Fast Multibeam Training for RIS-assisted Millimeter Wave Massive MIMO

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***Abstract*—For reconfigurable intelligent surface (RIS)-assisted mmWave massive MIMO, we propose a fast multibeam training (FMT) scheme with two stages. In the first stage, we find a multibeam together with the RIS reflection coefficients best matching the LoS links or the cascaded LoS links, where all the RISs use the same reflection coefficients at the same time to reduce the overhead of beam training. In the second stage, we find a DFT codeword best matching the LoS links or the cascaded LoS links for the BS instead of the multibeam. Different from the FMT scheme that uses a single-layer DFT codebook, in the proposed fast hierarchical multibeam training (FHMT) scheme we use a multi-layer hierarchical codebook, where in each layer only two wide beams are used to cover the angle space of interest, and the wide beams are narrowed down from top layer to bottom layer. Simulation results show that significantly smaller training overhead can be achieved by the FMT and FHMT schemes than the baseline scheme with slightly worse performance.**

***Index Terms*—Beam training, massive MIMO, millimeter wave (mmWave) communications, multibeam, reconfigurable intelligent surface (RIS).**

I. Introduction

Reconfigurable intelligent surface (RIS) has attracted the research interest due to its characteristics of low cost, energy efficient and lightweight. Specifically, RIS consists of a large number of elements that can be deployed for both the base station (BS) and the user equipment (UE). The deployment of the RIS introduces additional degrees of freedom for wireless channel, and therefore can create cascaded line-of-sight (LoS) links for UEs in the shadow area [1].

However, due to the large number of RIS elements, the acquisition of channel state information (CSI) is a challenging task. Two typical methods are channel estimation and beam training. Channel estimation is mainly to estimate the large-dimensional air-interface channel matrices, including those between any two among the BS, the RIS and the UE, by sending training symbols, pilot subcarriers or sounding beams [2]. However, the estimation of the high-dimensional cascaded channel matrix all the time requires many pilots overhead. Since the millimeter wave (mmWave) and terahertz MIMO channels have limited scattering paths, the high-dimensional cascaded channels can be estimated by exploiting the channel sparsity based on a compressed sensing algorithm [3]. In [4], the high-dimensional cascade channel is decomposed into the quasi-static BS-RIS channel and the low-dimensional dynamic RIS-UE channel to reduce pilot overhead for channel estimation. On the other hand, beam training can avoid the estimation of large-dimensional air-interface channel matrices, by transmitting sounding beams and then estimating the small-dimensional equivalent channel. Specifically, the BS, RIS and UE aim at finding the codeword best fitting for the wireless channels through the predefined codebooks. A straightforward approach for the beam training is to exhaustively test all the codewords predefined in the codebooks, which however results in large beam training overhead. To reduce the overhead, a multibeam training method is proposed, where the RIS reflection elements are divided into multiple sub-arrays to generate passive multibeam simultaneously pointing at different directions [5]. Then the investigation of the beam training is extended from the single RIS scenario to the multi-RIS scenario. A hierarchical beam training method with multi-RIS assisted based on partial search is proposed, where only the RIS codewords that satisfy the specific angle in the sine space need to be tested [6].

In this letter, we propose a fast multibeam training (FMT) scheme with two stages, where the second stage makes refinement of the results obtained from the first stage. In the first stage, we find a multibeam together with the RIS reflection coefficients best matching the LoS links or the cascaded LoS links, where all the RISs use the same reflection coefficients at the same time to reduce the overhead of beam training. In the second stage, we find a DFT codeword best matching the LoS links or the cascaded LoS links for the BS, by concentrating all the transmit power on it instead of the multibeam. Then we propose a fast hierarchical multibeam training (FHMT) scheme. Different from the FMT scheme that uses a single-layer DFT codebook to sweep the angle space of interest, in the FHMT scheme we use a multi-layer hierarchical codebook, where in each layer only two wide beams are used to cover the angle space of interest, and the wide beams are narrowed down from the top layer to the bottom layer. Note that different from the existing works that repeatedly perform the beam training with a different RIS [6], in this work we perform the beam training with all the RISs simultaneously. Different from [7] and [8] that enable simultaneous beam training for multiple UEs in a stage without consideration of the RIS, in this work two-stage beam training schemes are proposed for mmWave MIMO with multiple RISs, where in the first stage the beam training is simultaneously performed by the BS and multiple RISs so that the training process can be fast.

II. System Model

As shown in Fig. 1, we consider a downlink RIS-assisted mmWave MIMO communication system. The BS equipped with $N_{\rm BS}$ antennas serves a single-antenna UE, where the

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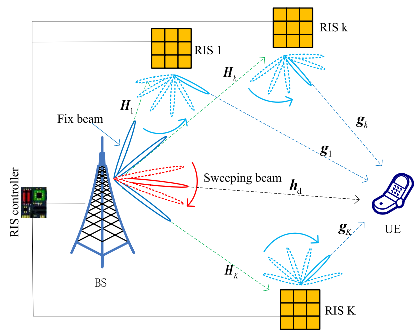

Fig. 1. A RIS-assisted communication system.

reflection coefficients of the K RISs are controlled by the BS. Suppose uniform linear arrays (ULAs) with half wavelength intervals are employed by the BS. For each RIS, the number of reflection elements is $N_{\rm R}$. Due to the severe path loss of mmWave wireless propagation, the signal reflected twice or more by the RISs can be ignored. According to the widely used Saleh-Valenzuela channel model, the channel matrix between the BS and the kth RIS, for $k=1,2,\dots,K$, is

$$\boldsymbol{H}_k=\sqrt{N_{\rm BS}N_{\rm R}/L_k^{\rm (b)}}\sum_{l=1}^{L_k^{\rm (b)}}\lambda_{k,l}^{\rm (b)}\boldsymbol{a}(N_{\rm R},\varphi_{{\rm R},k}^l)\boldsymbol{a}^{\rm H}(N_{\rm BS},\theta_{{\rm BS},k}^l), \tag{1}$$

where $L_k^{\rm (b)}$, $\lambda_{k,l}^{\rm (b)}$, $\varphi_{{\rm R},k}^l$ and $\theta_{{\rm BS},k}^l$ denote the number of multipath between the BS and the kth RIS, the complex-valued channel gain, the channel angle-of-arrival (AoA) and angle-of-departure (AoD) of the lth path for $l=1,2,\dots,L_k^{\rm (b)}$, respectively. The channel steering vector is defined as $\boldsymbol{a}(N,\theta)\triangleq[1,e^{j\pi\theta},\dots,e^{j(N-1)\pi\theta}]^{\rm T}/\sqrt{N}$, where N denotes the antenna number and θ is the channel AoA or AoD. Similarly, the channel vector between the BS and the UE can be given as $\boldsymbol{h}_{\rm d}=\sqrt{N_{\rm BS}/L^{\rm (d)}}\sum_{l=1}^{L^{\rm (d)}}\lambda_l^{\rm (d)}\boldsymbol{a}^{\rm H}(N_{\rm BS},\theta_{\rm BS}^l)$, where $L^{\rm (d)}$, $\lambda_l^{\rm (d)}$ and $\theta_{\rm BS}^l$ denote the number of multipath between the BS and the UE, the complex channel gain and the AoD of the lth multipath, $l=1,2,\dots,L^{\rm (d)}$, between the BS and the UE, respectively. The channel vector between the kth RIS and the UE can be given as $\boldsymbol{g}_k=\sqrt{N_{\rm R}/L_k^{\rm (r)}}\sum_{l=1}^{L_k^{\rm (r)}}\lambda_{k,l}^{\rm (r)}\boldsymbol{a}^{\rm H}(N_{\rm R},\theta_{{\rm R},k}^l),k=1,2,...,K$, where $L_k^{\rm (r)}$, $\lambda_{k,l}^{\rm (r)}$ and $\theta_{{\rm R},k}^l$ represent the number of multipath between the kth RIS and the UE, the complex channel gain and the AoD of the lth multipath, $l=1,2,\dots,L_k^{\rm (r)}$, between the kth RIS and the UE, respectively. In general, we assume that all the AoAs and AoDs obey uniform distribution and the channel gain follows a complex Gaussian distribution with zero mean and different variance.

We define $\mathcal{K}\triangleq\{1,2,\dots,K\}$. The function of the kth RIS for $k\in\mathcal{K}$ can be modeled as a diagonal matrix $\boldsymbol{\Theta}_k\triangleq{\rm diag}\left\{\beta_{k,1}e^{j\theta_{k,1}},\beta_{k,2}e^{j\theta_{k,2}},...,\beta_{k,N_{\rm R}}e^{j\theta_{k,N_{\rm R}}}\right\}$, where $\theta_{k,n}\in[-\pi,\pi]$ and $\beta_{k,n}\in[0,1]$ denote the phase and the gain of the nth reflection element, respectively. Since the RIS is usually placed in an open field of view such as the exterior surface of a building, the LoS link between the BS and the RIS usually exists. However, the UE is typically located in a complex environment and therefore the LoS links between the BS and the UE and between the RIS and the UE may not exist. Let $\mathcal{P}_{\rm d}$ and $\mathcal{P}_k$ represent the probabilities the LoS links between the BS and the UE and between the kth RIS and the UE are blocked the environment, respectively. The probability there exists a LoS link between the BS and the UE, including cascaded LoS links is $\mathcal{P}_{\rm T}=1-\mathcal{P}_{\rm d}\prod_{k=1}^{K}\mathcal{P}_k$. Compared with the scenario without any RIS, i.e., $\mathcal{P}_{\rm T}=1-\mathcal{P}_{\rm d}$, we can substantially improve $\mathcal{P}_{\rm T}$ by using the RISs, which implies the effectiveness of the RISs to enhance the signal coverage of mmWave MIMO.

Suppose the signal transmitted by the BS is denoted as x, where x is normalized by $\mathbb{E}\{|x|^2\}=1$. Then the signal received by the UE can be expressed as

$$y(\boldsymbol{w},\boldsymbol{\Theta}_1,\dots,\boldsymbol{\Theta}_K)=\sqrt{P}\big(\boldsymbol{h}_{\rm d}+\sum_{k=1}^{K}\boldsymbol{g}_k\boldsymbol{\Theta}_k\boldsymbol{H}_k\big)\boldsymbol{w}x+\eta. \tag{2}$$

where $\boldsymbol{w}$ and η denote the transmit beamforming vector at the BS satisfying $\|\boldsymbol{w}\|_2=1$, and the additional white Gaussian noise satisfying $\eta\sim\mathcal{CN}(0,\sigma^2)$, respectively. The total transmit power is P. Therefore, the achievable rate of the UE can be expressed as $R\triangleq\log_2(1+P|(\boldsymbol{h}_{\rm d}+\sum_{k=1}^K\boldsymbol{g}_k\boldsymbol{\Theta}_k\boldsymbol{H}_k)\boldsymbol{w}|^2/\sigma^2)$.

To efficiently generate $\boldsymbol{w}$ and $\boldsymbol{\Theta}_k$, we may establish codebooks for the BS and the RIS. The BS codebook $\mathcal{F}\in\mathbb{C}^{N_{\rm BS}\times N_{\rm BS}}$ can be denoted as $\mathcal{F}\triangleq\{\boldsymbol{f}(1),\boldsymbol{f}(2),\dots,\boldsymbol{f}(N_{\rm BS})\}$, which includes $N_{\rm BS}$ codewords, with the ith codeword being $\boldsymbol{f}(i)\in\mathbb{C}^{N_{\rm BS}}$ for $i\in\mathcal{I}$, where we define $\mathcal{I}\triangleq\{1,2,\dots,N_{\rm BS}\}$. Similarly, the codebook for each RIS $\mathcal{C}\in\mathbb{C}^{N_{\rm R}\times(N_{\rm BS}N_{\rm R})}$ can be denoted as $\mathcal{C}\triangleq\{\boldsymbol{c}(1),\boldsymbol{c}(2),\dots,\boldsymbol{c}(N_{\rm BS})\}$, which includes $N_{\rm BS}$ codewords, with the ith codeword being $\boldsymbol{c}(i)\in\mathbb{C}^{N_{\rm R}\times N_{\rm R}}$ for $i\in\mathcal{I}$. Note that we assume the number of codewords for each RIS the same as that for the BS so that the design of beam training schemes can be simplified.

We aim at maximizing the achievable rate of the UE by optimizing $\boldsymbol{w}$ and $\boldsymbol{\Theta}_k$, which can be expressed as

$$\max_{\boldsymbol{w},\{\boldsymbol{\Theta}_k,k\in\mathcal{K}\}} R, \quad {\rm s.t.}\quad \boldsymbol{w}\in\mathcal{F},\ \boldsymbol{\Theta}_k\in\mathcal{C},k\in\mathcal{K}. \tag{3}$$

However, in practice it is impossible to directly find $\boldsymbol{w}$ and $\boldsymbol{\Theta}_k$ from (3). Then we transform (3) into the following equivalent problem expressed as

$$\max_{\boldsymbol{w},\{\boldsymbol{\Theta}_k,k\in\mathcal{K}\}} |y(\boldsymbol{w},\boldsymbol{\Theta}_1,\boldsymbol{\Theta}_2,\dots,\boldsymbol{\Theta}_K)|, \tag{4a}$$

$$\quad{\rm s.t.}\quad \boldsymbol{w}\in\mathcal{F},\ \boldsymbol{\Theta}_k\in\mathcal{C},k\in\mathcal{K}. \tag{4b}$$

The most intuitive solution for (4) is to exhaustively test all codewords and then select the best one. However, the training overhead is $N_{\rm BS}^{K+1}$, which is very large in practice. In the following, we will design beam training schemes to reduce the overhead of resource.

III. Beam Training Design

A. Baseline Beam Training Scheme

First we power off all the RISs and perform the beam training for the BS and the UE. In this context, no RIS works. We test different BS codewords and compare the received signal power of the UE. From $N_{\rm BS}$ codewords, we select one achieving the largest power, which can be expressed as $\gamma_0=\arg\max_{i\in\mathcal{I}}|y(\boldsymbol{f}(i),\underbrace{\boldsymbol{0},\boldsymbol{0},\dots,\boldsymbol{0}}_{K})|^2$, where the subscript 0 of γ_0 means that no RIS works. Here we use the popular DFT codebook for $\mathcal{F}$. $\boldsymbol{f}(i)$ denotes the ith codeword in $\mathcal{F}$ and can be expressed as $\boldsymbol{f}(i)=\boldsymbol{a}(N_{\rm BS},-1+(2i-1)/N_{\rm BS}),\ i\in\mathcal{I}$.

To reduce the hardware complexity, in this letter no more than one RIS is powered on. Then we power on the kth RIS, while the other RISs are all powered off, for $k \in \mathcal{K}$. In this context, only one RIS works. Since the RISs are typically deployed before the wireless transmission, the position of the RISs is fixed and known to the BS. Moreover, the RISs are usually placed in an open field of view such as the exterior surface of a building. The LoS link between the BS and the RIS usually exists, where we denote the AoD of the LoS link as $\theta^1_{\mathrm{BS},k}$ according to (1). Suppose the BS codeword with the index γ_k achieves beam alignment with the kth RIS, i.e.,

$$\theta^1_{\mathrm{BS},k} \in \left[(-1+2\left(\gamma_k-1\right))/N_{\mathrm{BS}}, (-1+2\gamma_k)/N_{\mathrm{BS}}\right]. \tag{5}$$

To improve the fairness of the signal coverage, all the RISs are evenly deployed in space and γ_k is different from each other. We test different codewords for the kth RIS and compare the received signal power of the UE. From N_{BS} codewords, we select one achieving the largest power, which can be expressed as $i_k = \arg\max_{i\in\mathcal{I}} |y(\boldsymbol{f}(\gamma_k), \underbrace{\mathbf{0},\ldots,\mathbf{0}}_{k-1}, \boldsymbol{c}(i), \underbrace{\mathbf{0},\ldots,\mathbf{0}}_{K-k})|^2$. We also use the DFT codebook for $\mathcal{C}$. $\boldsymbol{c}(i)$ denotes the ith codeword in the $\mathcal{C}$ and can be expressed as $\boldsymbol{c}(i) = \mathrm{diag}\left\{\boldsymbol{a}(N_{\mathrm{R}}, -1+(2i-1)/N_{\mathrm{BS}})\right\}, i \in \mathcal{I}$.

After each RIS is powered on in turn, we have $K+1$ different received signal power measured at the UE, including K cases with K different RIS and one case with no RIS. From the $K+1$ cases, we select one achieving the largest received signal power, which can be expressed as

$$\begin{aligned}
m_{\mathrm{opt}} = \arg\max\nolimits_{m\in\{0,\mathcal{K}\}} \{&|y(\boldsymbol{f}(\gamma_0), \underbrace{\mathbf{0},\mathbf{0},\ldots,\mathbf{0}}_{K})|^2,\\
&|y(\boldsymbol{f}(\gamma_1), \boldsymbol{c}(i_1), \underbrace{\mathbf{0},\ldots,\mathbf{0}}_{K-1})|^2, |y(\boldsymbol{f}(\gamma_2), \mathbf{0}, \boldsymbol{c}(i_2), \underbrace{\mathbf{0},\ldots,\mathbf{0}}_{K-2})|^2,\\
&\cdots, \quad |y(\boldsymbol{f}(\gamma_m), \underbrace{\mathbf{0},\ldots,\mathbf{0}}_{m-1}, \boldsymbol{c}(i_m), \underbrace{\mathbf{0},\ldots,\mathbf{0}}_{K-m})|^2,\\
&\cdots, \quad |y(\boldsymbol{f}(\gamma_K), \underbrace{\mathbf{0},\ldots,\mathbf{0}}_{K-1}, \boldsymbol{c}(i_K))|^2\}.
\end{aligned}$$

Finally, the designed transmit beamformer is $\boldsymbol{w}_{\mathrm{opt}} = \boldsymbol{f}(\gamma_{m_{\mathrm{opt}}})$. If $m_{\mathrm{opt}} = 0$, all the RISs are powered off; otherwise, only the m_{opt}th RIS works and the reflection coefficients of that RIS are $\boldsymbol{\Theta}_{m_{\mathrm{opt}}} = \boldsymbol{c}(i_{m_{\mathrm{opt}}})$. In particular, the training overhead of this scheme is $(K+1)N_{\mathrm{BS}}$.

B. Fast Multibeam Training (FMT) Scheme

Since RISs are usually placed in an open field of view with a LoS link between the BS and RIS, $\{\theta^1_{\mathrm{BS},k},\ k \in \mathcal{K}\}$ is known before the beam training. We may simultaneously transmit K beams, including $\boldsymbol{f}(\gamma_1)$, $\boldsymbol{f}(\gamma_2)$, $\ldots$, $\boldsymbol{f}(\gamma_K)$, with $\boldsymbol{f}(\gamma_k)$ pointing at the kth RIS for $k \in \mathcal{K}$. In fact, these K beams form a multibeam, which are fixed during the beam training.

To determine the transmit beamformer for the UE, the BS needs to use sweeping beams. At each time slot, the sweeping beam points at a different direction that the UE may be located in, while at the UE the received signal power is measured. Once all available beams are tested, we compare the received signal power and select the largest one. Therefore, the fixed K beams and a sweeping beam forms a multibeam, which can be designed before the beam training and stored in $\mathcal{F}$.

To mitigate the inter-beam interference, we introduce K phase variables $\{\mu_k \in [0, 2\pi),\ k \in \mathcal{K}\}$. Then the transmit multibeam generated by K fixed beams and a sweeping beam in the ith time slot for $i \in \mathcal{I}$, can be expressed as

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Algorithm 1 FMT Scheme

Input: K, N_{BS}, N_{R}, $\mathcal{F}$ and $\mathcal{C}$.
Determine γ_k from (5).
for $i = 1, 2, \ldots, N_{\mathrm{BS}}$ **do**
  Determine $\boldsymbol{w}_i$ via (6), set $\boldsymbol{\Theta}_k = \boldsymbol{c}(i)$, for $k \in \mathcal{K}$.
  Get $y(\boldsymbol{w}_i, \boldsymbol{\Theta}_1, \boldsymbol{\Theta}_2, \ldots, \boldsymbol{\Theta}_K)$ from (2).
end for
Get i_{opt} from (7).
Power off all the RISs and set $\boldsymbol{w} = \boldsymbol{f}(i_{\mathrm{opt}})$.
Obtain z_0 via (8).
for $k = 1, 2, \ldots, K$ **do**
  Open the kth RIS only, set $\boldsymbol{w} = \boldsymbol{f}(\gamma_k)$,$\boldsymbol{\Theta}_k = \boldsymbol{c}(i_{\mathrm{opt}})$.
  Obtain z_k from (9).
end for
Get m_{opt} from (10).
Output: m_{opt}.

$$\boldsymbol{w}_i = \sum_{k=1}^{K} e^{j\mu_k} \boldsymbol{f}(\gamma_k) + \delta \boldsymbol{f}(i), \tag{6}$$

where $\delta \triangleq \begin{cases} 0, & \text{if } \ i \in \{\gamma_k,\ k \in \mathcal{K}\}, \\ 1, & \text{else.} \end{cases}$.

Note that we use δ to indicate if the UE is on the same direction of a RIS. If $\delta = 0$, we do not need to transmit any sweeping beam. The K phase variables can be optimized based on the existing methods [9], and finally the power of the designed transmit beamformer needs to be normalized.

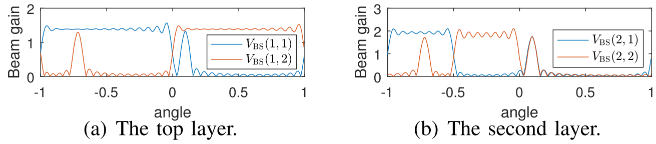
Once the multibeams are designed by (6), we begin the beam training. The proposed FMT scheme includes two stages, where the second stage makes a refinement for the results obtained from the first stage.

In the first stage, at the ith time slot, for $i \in \mathcal{I}$, the BS transmits $\boldsymbol{w}_i$, while each RIS uses $\boldsymbol{c}(i)$ as its reflection coefficients, i.e., $\boldsymbol{\Theta}_k = \boldsymbol{c}(i)$, for $k \in \mathcal{K}$. Then the UE measures the received signal power. From the N_{BS} measurement, we select one having the largest received signal power as

$$i_{\mathrm{opt}} = \arg\max_{i\in\mathcal{I}} |y(\boldsymbol{w}_i, \underbrace{\boldsymbol{c}(i), \boldsymbol{c}(i), \ldots, \boldsymbol{c}(i)}_{K})|^2 \tag{7}$$

where i_{opt} is the index of the time slot as well as the index of the codewords in $\mathcal{F}$ and $\mathcal{C}$ best matching the channel matrix. Note that all the K RISs use the same reflection coefficients at the same time slot to reduce the overhead of beam training. Since the channel gain of the LoS link is much larger than that of the non-line-of-sight (NLoS) link and there normally exists at least one LoS link, the transmit signal passing through the LoS links or the cascaded LoS links will lead to a large received signal power, no matter the K RISs use the same reflection coefficients or not. Therefore, the purpose of (7) is essentially to find a multibeam together with the RIS reflection coefficients best matching the LoS links or the cascaded LoS links, based on the largest received signal power. Note that if the RIS has a larger number of codewords than the BS, the BS will finish the beam sweeping earlier than the RIS and will wait until all the codewords are tested for the RIS.

To simplify the hardware complexity, no more than one RIS will be powered on. Therefore, in the second stage, we will find a DFT codeword best matching the LoS links or



(a) The top layer. (b) The second layer.

Fig. 2. Beam pattern of designed BS codebook for FHMT.

the cascaded LoS links for the BS, by concentrating all the transmit power on it instead of the multibeam.

First we power off all the RISs and use $\boldsymbol{f}(i_{\text{opt}})$ as the transmit beamformer for the BS, i.e., $\boldsymbol{w} = \boldsymbol{f}(i_{\text{opt}})$, where the received signal power by the UE is

$$z_0 \triangleq |y(\boldsymbol{f}(i_{\text{opt}}), \underbrace{\boldsymbol{0}, \boldsymbol{0}, \ldots, \boldsymbol{0}}_{K})|^2. \quad (8)$$

Then we power on the kth RIS, while the other RISs are powered off, for $k \in \mathcal{K}$. In this context, only one RIS works and it uses the reflection coefficients $\boldsymbol{c}(i_{\text{opt}})$, while the BS transmits $\boldsymbol{f}(\gamma_k)$, i.e., $\boldsymbol{w} = \boldsymbol{f}(\gamma_k)$ and $\boldsymbol{\Theta}_k = \boldsymbol{c}(i_{\text{opt}})$. The received signal power by the UE is

$$z_k \triangleq |y(\boldsymbol{f}(\gamma_k), \underbrace{\boldsymbol{0}, \ldots, \boldsymbol{0}}_{k-1}, \boldsymbol{c}(i_k), \underbrace{\boldsymbol{0}, \ldots, \boldsymbol{0}}_{K-k})|^2. \quad (9)$$

After each RIS is powered on in turn, we have $K+1$ different received signal power. We select the largest one by

$$m_{\text{opt}} = \arg \max_{m \in \{0, \mathcal{K}\}} z_m. \quad (10)$$

If $m_{\text{opt}} = 0$ meaning that all the RISs are powered off, the selected DFT codeword as the designed transmit beamformer is $\boldsymbol{f}(i_{\text{opt}})$; otherwise, only the m_{opt}th RIS works, where the selected DFT codeword as the designed transmit beamformer is $\boldsymbol{f}(\gamma_{m_{\text{opt}}})$ and the reflection coefficients of the RIS is $\boldsymbol{c}(i_{\text{opt}})$.

The detailed steps of the proposed FMT scheme are summarized in **Algorithm** 1. Note that the training overhead of this scheme is $(K+1) + N_{\text{BS}}$, which is much smaller than that of the baseline beam training scheme. In fact, the FMT scheme can be extended to the scenario with multiple UEs. For that scenario, the beam training in the first stage will be the same, while that in the second stage will be performed for each UE in time-division multiple access (TDMA) method.

C. Fast Hierarchical Multibeam Training (FHMT) Scheme

To further reduce the training overhead, inspired by the hierarchical beam training for the mmWave massive MIMO without RIS, we propose a FHMT scheme, which combines the hierarchical beam training and the multibeam training.

We keep $\{\boldsymbol{f}(\gamma_k), k \in \mathcal{K}\}$ as a part for all the codewords, where $\boldsymbol{f}(\gamma_k)$ points at the K RISs and never changes during the beam training. Different from the FMT scheme that uses a single-layer DFT codebook to sweep the angle space of interest, in this scheme we use a multi-layer hierarchical codebook, where in each layer only two wide beams are used to cover the angle space of interest. The wide beams are narrowed down from the top layer to the bottom layer. In fact, the codewords in the hierarchical codebook can be well designed using the generalized method such as the two-step codeword design method [10]. We denote the codebooks for the BS and the RISs by $\boldsymbol{V}_{\text{BS}}$ and $\boldsymbol{V}_{\text{R}}$, respectively. $\boldsymbol{V}_{\text{BS}}$ and $\boldsymbol{V}_{\text{R}}$ have the same number of layers and the same number of codewords per layer, where the ith codeword in the sth layer

Algorithm 2 FHMT Scheme

Input: $\boldsymbol{V}_{\text{BS}}, \boldsymbol{V}_{\text{R}}, I$.
for $s = 1, 2, \ldots, S$ **do**
  Set $\boldsymbol{w} = \boldsymbol{V}_{\text{BS}}(s, I), \boldsymbol{\Theta}_k = \boldsymbol{V}_{\text{R}}(s, I)$, for $k \in \mathcal{K}$.
  Get r_I from (2).
  Set $\boldsymbol{w} = \boldsymbol{V}_{\text{BS}}(s, I+1), \boldsymbol{\Theta}_k = \boldsymbol{V}_{\text{R}}(s, I+1)$ for $k \in \mathcal{K}$.
  Get r_{I+1} from (2).
  if $s \neq S$ **then**
    Update I via (11).
  else
    Update I via (12).
  end if
end for
Set $i_{\text{opt}} = I$ and get m_{opt} in **Algorithm** 1 Step 8 to 14.
Output: m_{opt}.

of $\boldsymbol{V}_{\text{BS}}$ and $\boldsymbol{V}_{\text{R}}$ for $s = 1, 2, \ldots, \log_2 N_{\text{BS}}$ and $l = 1, 2, \ldots, 2^s$ are denoted as $\boldsymbol{V}_{\text{BS}}(s, l)$ and $\boldsymbol{V}_{\text{R}}(s, l)$, respectively. Note that $\boldsymbol{V}_{\text{R}}$ is a standard hierarchical codebook instead of the hierarchical multibeam codebook.

As an example, Fig. 2 shows the beam pattern of the BS codewords in the top layer and the second layer, where $N_{\text{BS}} = 32$, $K = 2$, $\gamma_1 = 5$ and $\gamma_2 = 18$. The beam coverage of each codeword includes not only the angle space of interest but also the angle space of the DFT codewords whose indices are γ_1 and γ_2. Suppose $\boldsymbol{V}_{\text{BS}}(1, 2)$ in Fig. 2(a) is selected after the top-layer beam training. For the second layer, we use $\boldsymbol{V}_{\text{BS}}(2, 1)$ and $\boldsymbol{V}_{\text{BS}}(2, 2)$ in Fig. 2(b) for beam training.

Once the hierarchical codebooks are designed, we begin the beam training. The proposed FHMT is divided into two stages, where the second stage makes a refinement for the results obtained from the first stage. In the first stage, similar to the FMT scheme, we find a hierarchical multibeam together with the RIS reflection coefficients best matching the LoS links or the cascaded LoS links, based on the largest received signal power, where all the RISs use the same reflection coefficients at the same time to reduce the overhead of beam training.

In the first stage, the index of the multibeam to be tested via the beam training is defined by I, which is initialized to be $I = 1$. The hierarchical codebooks $\boldsymbol{V}_{\text{BS}}$ and $\boldsymbol{V}_{\text{R}}$, both with total $S = \log_2 N_{\text{BS}}$ layers can be divided into two parts.

1) *The top layer and the intermediate layers*: It includes the first layer to the $(S-1)$th layer of the codebook. In the sth layer, for $s = 1, 2, \ldots, S-1$, the BS firstly transmits $\boldsymbol{w} = \boldsymbol{V}_{\text{BS}}(s, I)$, while each RIS uses $\boldsymbol{\Theta}_k = \boldsymbol{V}_{\text{R}}(s, I)$, for $k \in \mathcal{K}$ as its reflection coefficients. Then the received signal by the UE can be denoted as $r_I \triangleq y(\boldsymbol{w}, \boldsymbol{\Theta}_1, \boldsymbol{\Theta}_2, \ldots, \boldsymbol{\Theta}_K)$. Then the BS transmits $\boldsymbol{w} = \boldsymbol{V}_{\text{BS}}(s, I+1)$, while each RIS uses $\boldsymbol{\Theta}_k = \boldsymbol{V}_{\text{R}}(s, I+1)$, for $k \in \mathcal{K}$ as its reflection coefficients and the received signal by the UE can be denoted as $r_{I+1} \triangleq y(\boldsymbol{w}, \boldsymbol{\Theta}_1, \boldsymbol{\Theta}_2, \ldots, \boldsymbol{\Theta}_K)$. By comparing the absolute value of r_I and r_{I+1}, we can update I by

$$I = \begin{cases} 2I-1, & \text{if } |r_I| \geq |r_{I+1}|, \\ 2I+1, & \text{else}. \end{cases} \quad (11)$$

We will test the multibeams in the next layer via the beam training using the updated I.

2) *The bottom layer*: It is the Sth layer of the codebook. The only difference from the first part is how to update I.

$$I = \begin{cases} I, & \text{if } |r_I| \geq |r_{I+1}|, \\ I+1, & \text{else}. \end{cases} \tag{12}$$

Then we finish the first stage. We set $i_{\text{opt}} = I$. The procedures in the second stage are exactly the same as Step 8 to Step 14 in **Algorithm** 1.

The detailed steps for the proposed FHMT scheme are summarized in **Algorithm** 2. The training overhead is $(K+1)+2\log_2 N_{\text{BS}}$ and there is a further reduction in the overhead.

IV. Simulation Results

We consider a RIS-assisted mmWave massive MIMO system, where the BS equipped with $N_{\text{BS}} = 32$ antennas serves a single-antenna UE, assisted by $K = 2$ RISs. The number of the reflection elements in each RIS is $N_{\text{R}} = 32$. The channel matrix between the BS and the kth RIS, and the channel vectors between the BS and the UE and between the kth RIS and the UE, for $k = 1, 2$ are modeled with 3 channel paths with one LoS path and two NLoS paths, where the channel gain of the LoS path obeys $\mathcal{CN}(0, 1)$ and that of two NLoS paths obeys $\mathcal{CN}(0, 0.01)$. The probabilities that the LoS links are blocked are $\mathcal{P}_{\text{d}} = \mathcal{P}_{\text{k}} = 0.3$. Once the LoS path is blocked, the channel gain of the LoS path is set to zero.

As shown in Fig. 3, we compare the success rate of beam training for the baseline beam training scheme, FMT scheme and FHMT scheme. The success rate is defined according to the convention [10]. If the DFT codewords best matching the LoS links or the cascaded LoS links can be correctly identified after the beam training, we define that the beam training is successful; otherwise, we define that the beam training is failed. The ratio of the number of successful beam training over the total number of beam training is defined as the success rate. It can be seen from Fig. 3 that in low signal-to-noise ratio (SNR) condition, the success rate of the baseline beam training scheme is higher than that of the other two schemes. The reason that the performance of FHMT deteriorates more heavily that that of FMT at low SNR, is that the wide beam is more sensitive to the noise than the narrow beam. Moreover, with the increase of SNR, the gap of the success rate between any beam training scheme we propose and the baseline beam training scheme is decreasing. In particular, when SNR is 18dB, the success rate of the FMT and FHMT schemes we propose is 2.7% and 5.0% lower than the baseline beam training, respectively. However, the training overhead of the FMT and FHMT schemes can be reduced by 63.5% and 86.5% compared to the baseline beam training, respectively.

As shown in Fig. 4, we compare the achievable rate for the baseline beam training scheme, FMT scheme and FHMT scheme. From the figure, the curves of all schemes increase almost linearly, and the performance gap between any beam training scheme we propose and the baseline beam training scheme is decreasing. In particular, when SNR = 0 dB, the performance of the FMT scheme and the FHMT scheme can reach 92.4% and 71.1% of that of the baseline beam training scheme, respectively; and when SNR = 20 dB, the achievable rate of the two schemes can reach 99.6% and 96.8% of that of the baseline beam training scheme, respectively. As

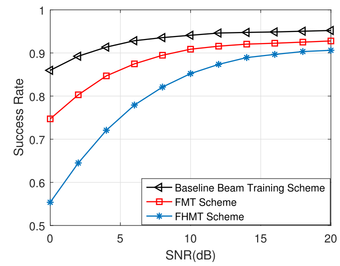


Fig. 3. Comparisons of success rate for different beam training schemes.

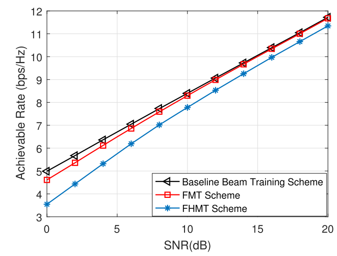


Fig. 4. Comparisons of achievable rate for different beam training schemes.

a conclusion, we can achieve significantly smaller training overhead by the FMT and FHMT schemes than the baseline beam training scheme, only with slightly worse performance in terms of the achievable rate.

V. Conclusion

In this paper, we have investigated the beam training for RIS-assisted mmWave massive MIMO. We have proposed the FMT and FHMT schemes with two stages. In the future, we will further consider the theoretical analysis for the beam training and investigate low-overhead beam training schemes.

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