A Novel Chaos Based Cost Function for Power Control of Cognitive Radio Networks

Ali Al Talabani, Arumugam Nallanathan, and Huan X. Nguyen

***Abstract*—The game theoretic approach has been recently considered as potential solutions for power control in cognitive radio network (CRNs). It is well known that a critical issue in applying game theory is the selection of its proper cost function. In this letter, we introduce a new chaos based cost function to design power control algorithm and analyse the dynamic spectrum sharing issue in the uplink of cellular CRNs. Particularly, for secondary users as the game players in underlay scenarios, we define utility/cost functions taking into account the interference from and the interference tolerance of the primary users. We prove the existence of the Nash equilibrium in this power control game, leading to significantly lower power consumption and relatively fast convergence rate when compared to the other existing game algorithms.**

***Index Terms*—Cognitive radio (CR) networks, power control, game theory, chaos.**

I. Introduction

THE RADIO spectrum resources have become more congested due to the explosive growth of wireless applications. This is a bottleneck problem which restricts the continuous development of wireless mobile communications and services. Cognitive radio (CR), a new developing intelligent radio technology based on software radio, is widely considered as a promising technology that addresses the spectrum shortage problem caused by the current inflexible spectrum-allocation policy. By sensing the external environment and studying the environment using artificial intelligent techniques, it detects the primary user (PU)'s presence, which allows the secondary user (SU) to take up the spectrum when available, and changes its transmission parameters to adapt to the changing environment [1].

The major goal of CR network (CRN) is to provide transmission opportunities and substantial quality of service (QoS) for SUs, and avoid harmful interference to the PUs. In this sense, power control is essential for CRNs. Recently, power control for CRNs has drawn considerable attention. Particularly, the topic of power control in traditional wireless networks has been studied extensively. Earlier schemes such as signal-to-noise plus interference ratio (SINR) balancing which was initially proposed for satellite communications and then adapted to wireless communications suffer from slow convergence [2]. The model in [3] proposed a cost for each mobile that consists of a weighted sum of power and square of SIR error and obtained the static Nash equilibrium for the resulting costs. Also, recent studies demonstrated the impact of the nature of channel knowledge on CR capacity, particularly focusing on the importance of channel state information (CSI) between the SU and the PU [8]. In [1], a new iterative algorithm was suggested using the game theory. It considered not only the SINR requirement, as other game theoretic algorithms, but also the influence of the power threshold. However, this algorithm suffers from the increase in power consumption. A new cost based primary-secondary interaction modelling in cellular cognitive radio networks was proposed in [9]. They proposed new utility and cost functions for primary and secondary users, respectively. Also, they improved convergence and power consumption of secondary users as users are given the opportunity to switch between different base stations. Since the characteristics of chaotic motion is described by ergodicity, randomization and regularity which can traverse all status of the chaos according to its own mapping function without repetition within a certain scope. Therefore, some research works have applied chaotic variables into the optimization search and showed that it is useful in enhancing the convergence of random search based optimization [4]. In this letter, a new game theory based power control algorithm is proposed for CRNs. In particular, the contributions of work are summarised as follows:

- A novel and efficient cost function based on chaos logistic map is proposed, which guarantees the convergence of the '*power*' game to a unique Nash equilibrium.
- The power control is achieved with a significant reduction of power consumption for the cognitive users (at least by half compared to other methods).
- It is also shown that the rate of convergence of the proposed chaotic algorithm is relatively fast compared to other existing iterative methods.

II. Power Control Based on Game Theory

It is shown that the game theory, a strong tool in economics, can also be used to solve the problem of power control in wireless communication systems [1]. Based on a suitable utility or cost function, the game solution can help obtain the power control policies through effective iterative algorithms.

A. System Model

The system model is shown in Fig. 1 in which a CR spectrum-sharing network includes a set of N cognitive SUs coexisting with the PUs, a primary base station (PBS) (serving the PUs), and a secondary base station (SBS) (serving the cognitive network).

Cognitive users use coexistence scenario to share spectrum resources with the primary users. Let g_i and h_i denote the link gains from the cognitive user i to the SBS and the PBS, respectively. Note that several SUs will interfere with the PUs

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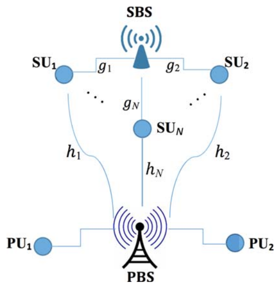

Fig. 1. Illustration of an uplink CR system model with N SUs and 2 PUs.

to a certain degree. The goal of power control is to limit SUs' power to avoid excessive interference. Particularly, we focus on the uplink power control problem. One of the designing goals of power control in wireless networks is to ensure that no mobile's SINR falls below its threshold γ_{tar} to ensure adequate QoS. In this letter, flat fading channel is assumed, where channel gains remain quasi-static over time. We assume that an CSI knowledge between the PUs and the SUs is known. The SINR of the ith SU signal received at the SBS is defined as

$$\gamma_i = \frac{p_i g_i}{\sum_{i \neq j} p_j q_{i,j} + \mu_i + \sigma_i} \tag{1}$$

where

$$\gamma_i \geqslant \gamma_{\text{tar}} \qquad \forall i \tag{2}$$

and σ_i is the background noise power; μ_i is interference power from the PUs affecting the ith SU signal at the SBS; p_i is the power level of the ith SU; and $q_{i,j} = g_j \delta_{i,j}$. Here, $\delta_{i,j}$ denotes the correlation between the ith SU's signal and the jth SU's signal. Thus, q_{ij} can be seen as an effective link gain from the jth user to SBS when considering its interference to the ith SU's signal.

To meet the user's QoS in CRNs, the power of the ith SU (p_i) should satisfy

$$\sum_{i=1}^{N} p_i h_i \leq T_{\max} \tag{3}$$

where $T_{\max}$ is the interference tolerance of PUs. But for a cognitive user, it should reach the minimum required SINR once the network allows it to communicate.

B. Game Theory and Problem Formulation

To enhance communication effect, a cognitive user needs to increase its SINR, which will frequently require high power. However, lower power is useful to decrease interference with other SUs. Therefore, low power and high SINR contradict each other. To resolve the conflict and maximize the benefit of SUs, we try to address this problem by using game theory. We assume that every SU is rational and wants to maximize his benefit, so we convert this problem to a non-cooperative game problem. In this section, we formulate the SINR-based power control problem as a non-cooperative game by choosing an appropriate cost function and finding the corresponding Nash equilibrium power vector. The power control game is defined as: $\mathcal{G} = [\mathcal{N}, \mathcal{P}, J]$, where now $\mathcal{N}$ is the set of N players (SUs), $\mathcal{P}$ is the strategy (power) set for the SUs, and J is the set of cost functions. We denote $J(p_i, \gamma_i(\mathbf{p}))$ as the cost function of the ith SU, where $\mathbf{p} = [p_1, p_2, \cdots, p_N]$[3]. The generalized Nash equilibrium problem where each player selfishly optimizes his own well-being within his strategy set that also depends on the strategies of the other players is defined as follows

$$\mathcal{G} : \min J(p_i, \gamma_i(\mathbf{p})), \qquad \forall i = 1, 2, \ldots, N \tag{4a}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} p_i h_i \leq T_{\max}, \tag{4b}$$

$$\gamma_i(\mathbf{p}) \geqslant \gamma_{\text{tar}}. \tag{4c}$$

The corresponding Nash equilibrium strategy is represented by the power vector $\mathbf{p}^* = [p_1^*, p_2^*, \cdots, p_N^*]$, where p_i^* is the Nash power, having the property that no individual user can lower its cost by deviating from p_i^*, i.e,

$$J(p_i^*, \gamma_i(\mathbf{p}^*)) \leq J(p_i, \gamma_i(\mathbf{p}_i^*)). \tag{5}$$

Here, $\mathbf{p}_i^* = [p_1^*, p_2^*, \cdots, p_{i-1}^*, p_i, p_{i+1}^*, \cdots, p_N^*]$. Every player (i.e., cognitive user) faces such a situation that its present strategy is optimal when other players do not change their strategies, i.e., the game achieves the Nash equilibrium. Thus, our power control algorithm will search for the Nash equilibrium point (i.e., an transmitting power level) to make the user's utility maximal (i.e., maximal transmission rate of information). Note that there are two conflicting objectives. Generally we target higher SINR for better service. However, higher SINR is achieved at the cost of increased drain on the battery and higher interference to signals of other users. Therefore, we define a cost function for each user depending on both power and SINR. Particularly, we consider the cost of the difference between the actual SINR and the target SINR that is chosen based on the estimated frame error rate. To ensure non-negativity and convexity of the cost function (i.e., to allow the existence of a nonnegative minimum), we use the squared SINR error term. Furthermore, the basic idea of chaotic optimization is mapping the chaotic variable into an optimized variable space, and then searching for the global optimum using its ergodicity of chaos movement [4]. Therefore, we include chaotic variable in our cost function to minimize power consumption to an acceptable level. Based on the SINR constraint in (2), we construct the following cost function:

$$J(p_i, \gamma_i) = a p_i^2 + 2\Delta_{\text{tar},i}\gamma_i + c(\gamma_{\text{tar}} - \gamma_i)^2 \tag{6}$$

where a and c are constants; $\Delta_{\text{tar},i}$ is an acceptable error level of the target SINR which is controlled by chaotic variable to allow trade-off between p_i and γ_i. Obviously, for any non-negative a and c, $J(p_i, \gamma_i)$ is a convex function with respect to p_i. Therefore, Nash equilibrium for our power control problem always exists. To derive Nash equilibrium, we set the partial derivative $\partial J / \partial p_i = 0$. Rearranging terms yields

$$\frac{\partial J}{\partial p_i} = 2a p_i + 2\Delta_{\text{tar},i}\frac{g_i}{I_i} + 2c(\gamma_{\text{tar}} - \gamma_i)\left(-\frac{g_i}{I_i}\right) = 0$$

where $I_i = \sum_{i \neq j} p_j q_{ij} + \mu_i + \sigma_i$. Then, by substituting $I_i / g_i = p_i / \gamma_i$, we obtain

$$\gamma_i = (\gamma_{\text{tar}} - \Delta_{\text{tar},i}) - \frac{a p_i^2}{c\gamma_i}, \tag{7}$$

$$\Rightarrow p_i = (\gamma_{\text{tar}} - \Delta_{\text{tar},i})\frac{p_i}{\gamma_i} - \frac{a p_i^2}{c\gamma_i}\left(\frac{p_i}{\gamma_i}\right). \tag{8}$$

Obviously, (8) can be used to obtain p_i^* through iterations as follows

$$p_i^{(l+1)} = \left(\gamma_{\text{tar}} - \Delta_{\text{tar},i}^{(l)}\right)\frac{p_i^{(l)}}{\gamma_i^{(l)}} - \frac{a\left(p_i^{(l)}\right)^3}{c\left(\gamma_i^{(l)}\right)^2} \triangleq f\left(p_i^{(l)}\right) \tag{9}$$

where superscript $(\cdot)^{(l)}$ denotes the lth iteration and $\Delta_{\text{tar},i}^{(l)}$ is updated by logistic map with chaotic variable $ch_i^{(l)}$ as below

$$ch_i^{(l+1)} = 4ch_i^{(l)}\left(1 - ch_i^{(l)}\right), \tag{10}$$

$$\Delta_{\text{tar},i}^{(l+1)} = (\gamma_{\text{tar}} - \epsilon\gamma_{\text{tar}})ch_i^{(l+1)}. \tag{11}$$

Here, ϵ is a parameter with value chosen close to 1 (we choose $\epsilon \in [0.97, 0.99]$ in our design, i.e., a drift of 1% to 3% from the target SINR γ_{tar} is acceptable) while the initial $ch_i^{(0)}$ has a value chosen in $[0,1]$.

C. Convergence to Unique Nash Equilibrium

Since Nash equilibrium is a fixed point of the best response functions, the existence of at least one Nash equilibrium point is guaranteed by a proper choice for the cost function in (5). It has been shown that if a fixed point of the algorithm $p_i^{(l+1)} = f(p_i^{(l)})$ exists and if the function f satisfies three properties—positivity, monotonicity, and scalability - then the power control algorithm converge to a unique Nash equilibrium. In the following, we will prove that these three properties are satisfied in our proposed algorithm:

1) *Positivity*: $f(p_i) > 0$. This is easily obtained from (9) based on the fact that we can choose $a/c \ll 1$ and the fact that $\gamma_{\text{tar}} \gg \Delta_{\text{tar},i}$ as seen from (11).
2) *Monotonicity*: $p_i > p_i'$ then $f(p_i) > f(p_i')$. To prove this property, we consider the term $f_i(p) - f_i(p')$ that is equal to
$$\frac{\gamma_{\text{tar}} - \Delta_{\text{tar},i}}{g_i}\left(I_i - I_i'\right) - \frac{a}{cg_i^2}\left(p_i I_i^2 - p_i' {I_i'}^2\right).$$
Since $p_i > p_i'$ then $I_i > I_i'$. Also, we already assume $a/c \ll 1$ then $a/(cg_i^2) \ll (\gamma_{\text{tar}} - \Delta_{\text{tar},i})/g_i$. Thus, $f_i(p_i) - f_i(p_i')$ would be a positive value.
3) *Scalability*: $f(\alpha p_i) < \alpha f(p_i) \forall \alpha > 1$. If this condition is satisfied, the algorithm converges to a unique fixed point. From (9), we obtain
$$\alpha f(p_i) - f(\alpha p_i) = \frac{ap_i^3}{c\gamma^2}(\alpha^3 - \alpha). \tag{12}$$
It is obvious that from (12), the *scalability* condition is satisfied due to $\alpha > 1$.

Positivity and monotonicity of $f(p_i)$ impose constraints on acceptable values of I_i but scalability restricts the allowable receiver noise power level and generates a limit less than that required for monotonicity.

D. Iterative Chaos Based Power Control Algorithm

The procedure of applying chaotic iterative algorithm of power control is as follows:

1) *Step 1*: Set the target SINR (γ_{tar} and the interference tolerance of PUs, $T_{\max}$. Also, obtain the channel matrix with coefficients g_i and q_{ij}. Set the initial value $ch_i^{(0)}$ in $[0,1]$. Set $l = 0$.
2) *Step 2*: Increase $l = l + 1$. Compute $ch_i^{(l)}$ and $\Delta_{\text{tar},i}^{(l)}$ by using (10) and (11), respectively.
3) *Step 3*: Apply (9) to calculate $p_i^{(l)}$ and then compute SINR $(\gamma_i^{(l)})$ according to (1).
4) *Step 4*}: If $\gamma_i^{(l)} < \epsilon\gamma_{\text{tar}}$ then return to Step 2. Otherwise, stop chaotic search by taking $ch_i^{(l+1)} = ch_i^{(l)}$ and then take $\Delta_{\text{tar},i}^{(l+1)} = \Delta_{\text{tar},i}^{(l)}$ and $p_i^{(l+1)} = p_i^{(l)}$.

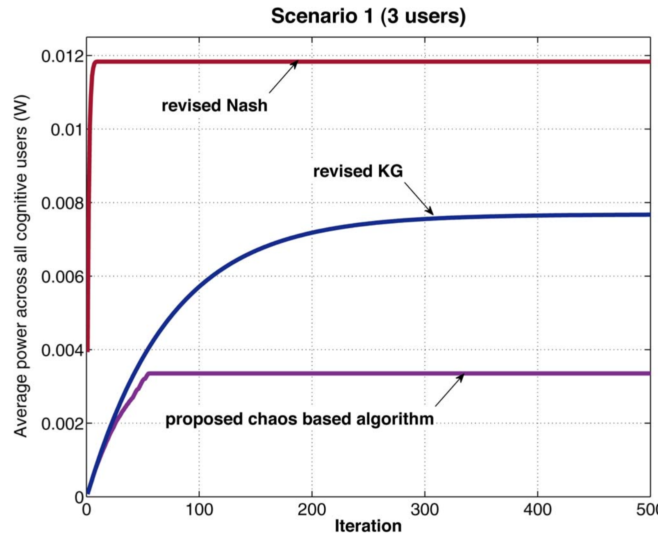

Fig. 2. Average power consumption in Scenario 1.

III. Results and Discussion

In this study, three different underlay scenarios are considered to study the effects of interference of PUs on the cognitive users while preserving interference tolerance of PUs (all three scenarios use 2 PUs): i) Scenario 1: Three SUs; ii) Scenario 2: Five SUs; and iii) Scenario 3: Ten SUs. The channel coefficient matrix **G** for the two scenarios are generated based on the on the locations of PUs and SUs with respect to base stations and the cross correlation between users signals. In particular, channel gain g_i is generated by the attenuation model $g_i = A/d^n$ where d is the distance between the user and the base station, which is uniformly randomly generated. Here, $\{A,n\} = \{10^{-4}, 3.5\}$ for Scenario 2 and $\{10^{-8}, 4\}$ for Scenario 3. However, in Scenario 1, we used the same fixed channel model as in [3] for comparison purpose, in which the channel matrix is

$$\mathbf{G} = \begin{pmatrix} 1 & 0.0882 & 0.0375 \\ 0.1524 & 0.95 & 0.3501 \\ 0.0767 & 0.0244 & 0.99 \end{pmatrix}. \tag{13}$$

A comparison is performed between the proposed system with the existing algorithms: the revised Nash algorithm [1], [5] (see (14)), the power balancing by Koskie and Gajic (KG) [3] (see (15)) and the revised KG [7], [9] (see (16)). Their iterative algorithms are as follows

$$p_i^{(l+1)} = (\gamma_{\text{tar}})\frac{p_i^{(l)}}{\gamma_i^{(l)}} + \frac{a\left(T_{\max} - p_i^{(l)}\right)}{c} \tag{14}$$

$$p_i^{(l+1)} = (\gamma_{\text{tar}})\frac{p_i^{(l)}}{\gamma_i^{(l)}} - \left(\frac{a{p_i^{(l)}}^2}{c{\gamma_i^{(l)}}^2}\right) \tag{15}$$

$$p_i^{(l+1)} = (\gamma_{\text{tar}})\frac{p_i^{(l)}}{\gamma_i^{(l)}} - \frac{p_i^{(l)}}{\gamma_i^{(l)}}\sinh^{-1}\left(\frac{a{p_i^{(l)}}^2}{c{\gamma_i^{(l)}}^2}\right), \tag{16}$$

where $a/c = 0.25$ for all algorithms. As the KG and revised KG algorithms are similar approaches, only the results of the revised KG algorithm are displayed. The system parameters are set as follows: $\gamma_{\text{tar}} = 5$ in both scenarios; $T_{\max} = 35$ mW, $\epsilon = 0.99$ for Scenario 1 while Scenario 2 and 3 are with $T_{\max} = 3.5$ mW, $\epsilon = 0.98$.

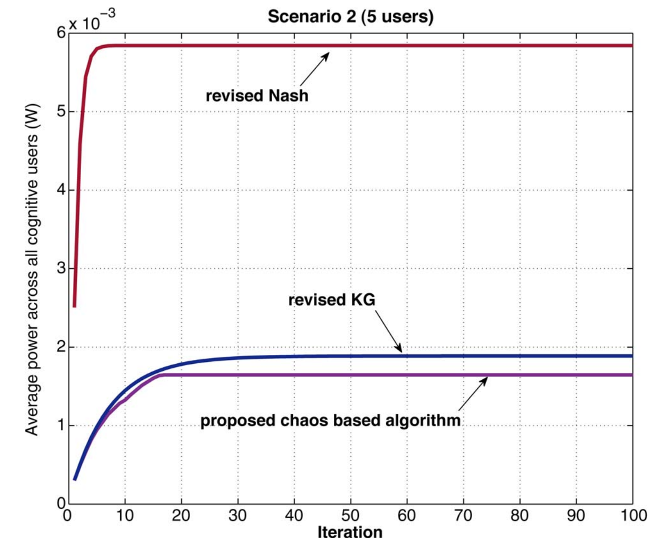

Fig. 3. Average power consumption in Scenario 2.

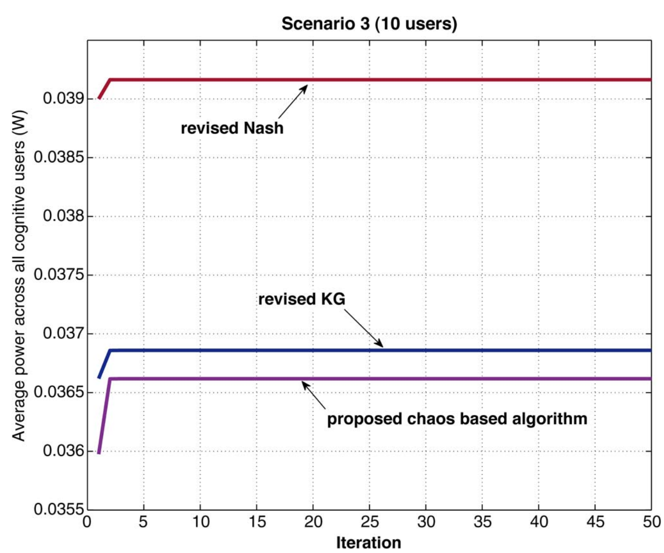

Fig. 4. Average power consumption in Scenario 3.

Fig. 2 shows the comparison of average power consumption across 3 SUs in Scenario 1. It is obvious that the power consumption of the proposed algorithm is less than that of the revised KG algorithm and particularly of the revised Nash algorithm. In addition, it converges after around 55 iterations, much faster than the revised KG algorithm (after 450 iterations) while only slightly slower than the revised Nash algorithm (after 15 iterations). Figs. 3 and 4 show the same performance behaviour with more number of cognitive users (5 SUs and 10 SUs, respectively) in Scenario 2 and 3. However, in these scenarios, the convergence rate tends to be faster for all algorithms when more users are involved.

Since the proposed algorithm is applied in underlay scenario, it is necessary to study the effect of variation of PUs' interference on SUs. Fig. 5 shows the impact of the interference from the PUs on the average SINR across the cognitive users. Obviously, when the interference level increases, it is expected that the SINR will reduce. However, we observe that the SINR of the proposed chaos based algorithm changes more slowly

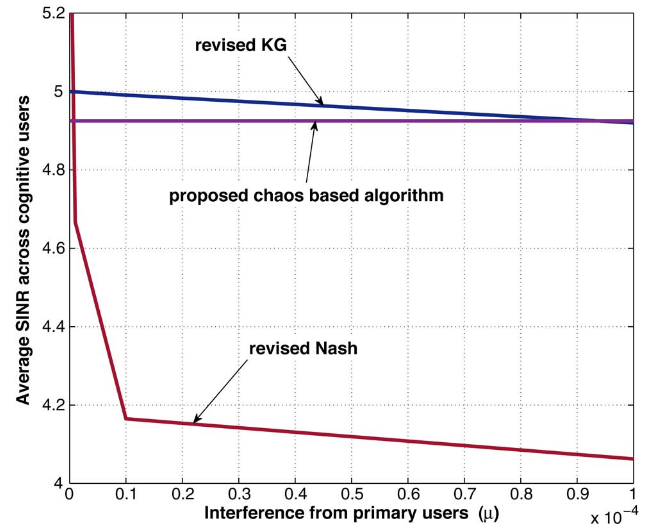

Fig. 5. Average SINR v. interference of PUs for Scen. 2.

(i.e., remains almost stable under the changing interference environment) compared with the other two algorithms. In addition, although the SINR of the proposed algorithm is slight smaller than that of the revised KG algorithm, it is generally significantly better than the revised Nash approach.

IV. Conclusion

In this letter, a novel chaotic cost function for power control based on non-cooperative game is proposed for CRNs. A resultant Nash equilibrium is achieved, which is shown to be unique in the coexistence scenario of PUs and SUs in CR systems. The simulation results show that the proposed algorithm achieves lowest power consumption at the expense of small drift (1–3%) from the target SINR in comparison with other existing game algorithms, while having a relatively fast convergence rate.

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