Outage Probability of Multiuser Relay Networks in Nakagami-m Fading Channels

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***Abstract*—We evaluate the performance of downlink multiuser relay networks (MRNs) equipped with a single amplify-and-forward (AaF) relay. A thorough and exact analysis is conducted to analyze the outage probability of MRNs under dissimilar Nakagami-m fading conditions. More specifically, we derive new closed-form expressions for the outage probability and the probability density function (pdf) of the highest end-to-end signal-to-noise ratio (SNR) associated with the strongest destination with the single user and Rayleigh fading as special cases. In particular, we provide new results for channel-state information (CSI)-based-gain relaying and fixed-gain relaying. We then demonstrate that the achievable diversity order is equal to either the first-hop fading parameter or the product of the second-hop fading parameter and the number of destinations. Furthermore, we derive compact closed-form expressions for the moments of the highest end-to-end SNR, from which other moment-based measures such as the average SNR and the amount of fading are deduced. Our results highlight the performance improvements offered by opportunistic scheduling and reveal the impact of the relay location with unbalanced hops on the overall performance. Various numerical examples illustrate the proposed analysis.**

***Index Terms*—Cooperative transmission, Nakagami fading, opportunistic scheduling.**

I. Introduction

RELAY deployment in wireless networks has been recognized as one of the core ingredients in the promotion of robust high-speed data transmission over challenging wireless environments [1], [2]. By placing a relay between the source and the destination, it is possible to provide higher transmission rates with broader coverage. Since the basic idea behind relaying is to assign a dedicated relay to help forward signals from the source to the destination, relay networks are also interpreted as cooperative communications.

Traditionally, the performance analysis of relaying has been viewed in the context of point-to-point links having a single source and a single destination. However, future broadband wireless networks are expected to provide ubiquitous high-speed information transfer to a large number of destinations (i.e., mobile users) at a reasonable cost and quality of service. In such point-to-multipoint links, the desired data rate may not be allowed if the destinations are in the far-off distance from the access point or in locations with severe shadowing. Therefore, good link quality and high-data-rate (HDR) coverage in large areas should be provided by exploiting novel cooperative network architectures. Hence, the objective of this paper is to study the joint interaction of multiuser transmission and relaying by quantifying the performance of cooperative multiuser networks in terms of the outage probability.

Among various cooperation protocols proposed in the open literature [3], we are concerned with the amplify-and-forward (AaF) relaying due to its low complexity and cost-effectiveness. AaF can be separated into two main categories: 1) channel-state information (CSI)-based-gain relaying, which adopts channel measurements to regulate a constant instantaneous transmit power at the relay; and 2) fixed-gain relaying, which simply applies a constant scaling gain to the received signal to regulate a constant average transmit power at the relay. Although CSI-based-gain relaying offers additional performance gains relative to fixed-gain relaying, fixed-gain relaying has become attractive from a cost-conscious and hardware-implementation point of view, due to its simplicity and ease of deployment. In this paper, we consider both AaF categories.

Driven by the potential application of such wireless relays, an upsurge of interest has been observed in studying the end-to-end performance of relays in various network configurations. The outage probability is an effective performance measure, and some pioneering studies have attempted to find closed-form solutions for this measure in *point-to-point dual-hop links* (i.e., a single source communicates with a single destination assisted by a single relay), as illustrated in Fig. 1(a). Among them, [4] proposed a semianalytical approach for the evaluation of the outage probability of dual-hop systems with CSI-based-gain relays over Nakagami fading channels. In [5], expressions were obtained for the statistics of the end-to-end signal-to-noise ratio (SNR) of dual-hop systems with CSI-based-gain relays over Rayleigh fading channels. The same authors in [6] studied the outage probability and the error performance of dual-hop systems for fixed-gain relays over Rayleigh fading channels. Closed-form upper bounds for the distributions of the

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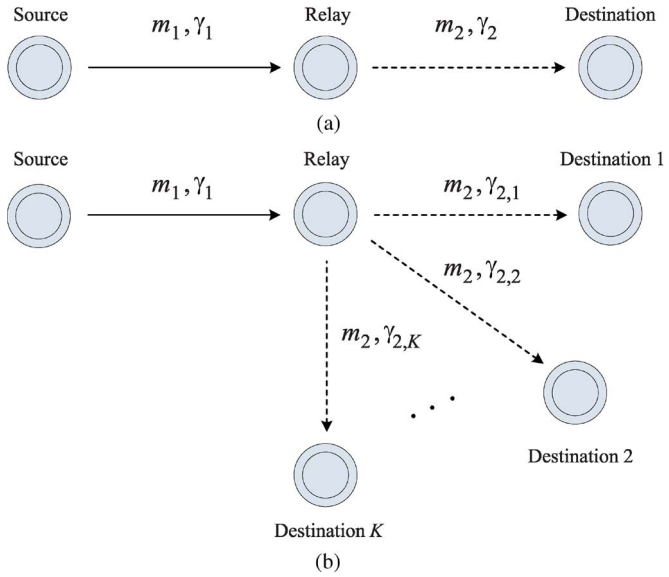

Fig. 1. Schematic of a dual-hop link, where there is practically no direct link between the source and the destination(s). (a) *Single user*: point-to-point dual-hop link. (b) *Multiuser*: point-to-multipoint dual-hop link (i.e., MRN).

end-to-end SNR of dual-hop systems with CSI-based- and fixed-gain relays over Nakagami fading channels were derived in [7]. A useful framework was outlined in [8] for obtaining the exact outage probabilities of dual-hop systems with CSI-based- and fixed-gain relays over Nakagami fading channels.

In a multiuser cellular wireless network, different mobile users sharing the radio channel are liable to experience different channel conditions at the same time. In such point-to-multipoint applications, the limited available radio resources must be allocated among all mobile users in the most efficient manner. Recently, opportunistic scheduling has attracted much attention as a scheduling method, in which the time-varying nature of a wireless channel is captured. Although all users share the common downlink channel, only one user is scheduled to transmit data at a time in a time slot. Intuitively, serving the user with the best channel conditions is expected to maximize the throughput and improve the system performance of the single-antenna broadcast channel [9]. This effect obtained with such scheduling is referred to as multiuser diversity [10], [11].

In many current systems, a plurality of remote destinations (or equivalently mobile users) may be included in geographically dispersed areas, leading to poor non-line-of-sight (NLOS) performance. Hence, direct transmissions between the base station and the destinations close to the cell boundary may not guarantee reliable communication at high data rates. In this regard, the key concept of cooperative transmission is applied to multiuser cellular wireless networks to assist transmissions between both ends. Introducing a relay in such a situation is a promising approach toward the potential improvements in channel quality by overcoming significant loss of signal strength along the propagation path in NLOS environments. In addition, introducing a relay in existing networks is considered to be an attractive means to extend the network coverage and/or capacity distribution. This concept is an example of *point-to-multipoint dual-hop links* (i.e., a single source communicates with multiple destinations assisted by a single relay), and we refer to this application as multiuser relay networks (MRNs), as illustrated in Fig. 1(b). Deployment of relays in multiuser cellular wireless networks is currently a promising option for emerging standards such as the IEEE 802.16j mobile multihop relay (MMR) network [12] and the wireless world initiative new radio (WINNER) [13].

Although the performance of point-to-point dual-hop links is well understood, knowledge about the impact of relays in a multiuser or network context is limited. Currently, very few works have analytically evaluated the performance of MRNs. Among them, [14] derived closed-form upper bound solutions for the outage capacities of a single fixed-gain relay. Through an approximation of the SNR distribution, [15] derived the average throughput of a single CSI-based-gain relay. On the other hand, some efforts in [16] investigated the outage probability of a single CSI-based-gain relay.

While the aforementioned works assumed Rayleigh fading, the performance of MRNs over generalized Nakagami-m fading has not been investigated. It is well known that Nakagami-m fading covers a wide range of fading scenarios that are typical in realistic wireless relay applications via the m parameter, which includes the Rayleigh fading $(m = 1)$ as a special case. Moreover, the effect of unbalanced hops on MRNs has not been thoroughly studied yet. In practice, the radio link between the source and the relay typically exhibits good line-of-sight (LOS) conditions, which is a realistic assumption for a scenario where both terminals are positioned above the rooftops, while the links between the relay and the destinations suffer severe multipath in highly congested built-up areas.

In this paper, we focus on the realistic potential gains of MRNs with AaF relaying in unbalanced generalized Nakagami-m fading conditions. In doing so, new expressions are derived for the cumulative distribution function (cdf) and the probability density function (pdf) of the highest end-to-end SNR associated with the *strongest* destination. The primary contributions of this paper are distinct from the previous works in the following aspects:

1) We present a unified analytical framework for the exact performance evaluation of MRNs based upon the direct implementation of the statistics of the per-hop SNR. The novelty of the proposed framework lies in the fact that we avoid the serious analytical difficulty arising from complicated Laplace transforms associated with the moment-generating function (mgf)-based approach.
2) We derive exact closed-form expressions for the outage probability of MRNs equipped with a CSI-based-gain relay. This scenario has previously been studied in [16] for the special case of Rayleigh fading but not the more generalized Nakagami-m fading.
3) We derive exact closed-form expressions for the outage probability of MRNs equipped with a fixed-gain relay. This scenario has never been addressed in previous works for either Rayleigh fading or Nakagami-m fading.
4) We derive the achievable diversity order of MRNs. This result proves that both CSI-based- and fixed-gain relaying yield the same diversity order, which is equal to either the

first-hop fading parameter or the product of the second-hop fading parameter and the number of destinations.

5) We derive exact closed-form expressions for the moments of the highest end-to-end SNR. Based on these new statistical results, we evaluate other relevant statistics that characterize the SNR distribution, such as the average SNR and the amount of fading (AoF).

Our new analytical expressions have the additional merit of characterizing the single-user point-to-point dual-hop scenario as a special case. Moreover, our generalized analysis serves as a tool to examine the impact of unbalanced hops by taking into account different per-hop average faded SNRs and/or different per-hop fading severities on the system performance. Furthermore, a design criterion is presented to guarantee performance improvements offered by opportunistic scheduling. Specifically, by varying the per-hop transmission distances, our results reveal the role of the relay location in determining the multiuser advantage.

II. System Description

A. Cooperative Transmission and Channel Model

Consider the multiuser wireless relay communication system depicted in Fig. 1(b). The source communicates with K destinations via the cooperative link with the aid of a single AaF relay. The direct link between the source and the destinations is absent. The relay transmission is constrained to a half-duplex mode, by dividing the transmission period into two consecutive signaling intervals. In signaling interval 1, the source transmits the signal to the relay; in signaling interval 2, the relay amplifies the received signal with a scaling gain factor and then forwards the scaled replica toward the destination that has the most favorable end-to-end channel quality.

We denote the average symbol energies available at the source and the relay as E_{S} and E_{R}, respectively. Furthermore, let the modulated signal transmitted by the source be denoted as x. In signaling interval 1, the received signal at the relay from the source can be represented as

$$y_r = \sqrt{E_{\mathrm{S}} d_{\mathrm{SR}}^{-\eta}}\alpha x + n_r \tag{1}$$

where α denotes the channel complex fading coefficient between the source and the relay, and n_r is the additive white Gaussian noise (AWGN) component with variance N_0 at the relay. The path loss is incorporated in the signal propagation, where d_{SR} is the distance between the source and the relay, and η is the path loss exponent. We define G as the scaling gain applied at the relay. Hence, in signaling interval 2, the received signal at the kth destination from the relay is expressed as

$$y_d = \sqrt{E_{\mathrm{R}} d_{\mathrm{RD}}^{-\eta}}\beta_k G\left(\sqrt{E_{\mathrm{S}} d_{\mathrm{SR}}^{-\eta}}\alpha x + n_r\right) + n_{d_k} \tag{2}$$

where β_k denotes the channel complex fading coefficient between the relay and the kth destination, n_{d_k} is the AWGN component with variance N_0 at the kth destination, and d_{RD} is the distance between the relay and the destination. Accordingly, the equivalent instantaneous end-to-end SNR of the kth destination $\gamma_{\mathrm{eq},k}$ can be obtained from (2) as

$$\gamma_{\mathrm{eq},k} = \frac{\gamma_1\gamma_{2,k}}{\gamma_{2,k} + \frac{1}{G^2 N_0}} \tag{3}$$

where $\gamma_1 = |\alpha|^2 d_{\mathrm{SR}}^{-\eta} E_{\mathrm{S}}/N_0$, and $\gamma_{2,k} = |\beta_k|^2 d_{\mathrm{RD}}^{-\eta} E_{\mathrm{R}}/N_0$ are the instantaneous faded SNRs of the first and second hop, respectively. From (3), we note that $\gamma_{\mathrm{eq},k}$ is influenced by the choice of G.

The pdf of the per-hop instantaneous faded SNR $Z = \{\gamma_1, \gamma_{2,k}\}$ follows a gamma distribution, i.e.,

$$f_Z(\gamma) = \frac{m_i^{m_i}\gamma^{m_i-1}}{\overline{\gamma}_i^{m_i}\Gamma(m_i)} e^{-\frac{m_i}{\overline{\gamma}_i}\gamma} \tag{4}$$

where $\Gamma(\cdot)$ is the gamma function defined by [17, eq. (8.310.1)] as $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$. The average faded SNR of the first and second hops is given by $\overline{\gamma}_1 = \mathbf{E}[\gamma_1]$ and $\overline{\gamma}_2 = \mathbf{E}[\gamma_{2,k}]$, respectively, where $\mathbf{E}[\cdot]$ is the expectation. The fading parameter of the first and second hops is denoted as m_1 and m_2, respectively.

It is assumed that the destinations are located within the same fading environment. In such a homogeneous environment, all the destinations have the same per-hop fading severity m_2 and per-hop average faded SNR $\overline{\gamma}_2$. On the other hand, the dual-hop transmission is assumed unbalanced, where the first and second hops experience different per-hop average faded SNRs (i.e., $\overline{\gamma}_1 \neq \overline{\gamma}_2$) and/or different per-hop fading severities (i.e., $m_1 \neq m_2$). This is typical in most relay applications, where the relay and the destinations are located in different fading environments.

The corresponding cdf of Z can be written as

$$F_Z(\gamma) = \frac{\gamma\left(m_i, \frac{m_i}{\overline{\gamma}_i}\gamma\right)}{\Gamma(m_i)} = 1 - \frac{\Gamma\left(m_i, \frac{m_i}{\overline{\gamma}_i}\gamma\right)}{\Gamma(m_i)} \tag{5}$$

where $\gamma(\cdot,\cdot)$ is the lower incomplete gamma function defined by [17, eq. (8.350.1)], and $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function defined by [17, eq. (8.350.2)].

B. Opportunistic Scheduling in Cooperative Transmission

Time-division multiple access (TDMA) is assumed to facilitate the sharing of the downlink channel among the multiple destinations. Specifically, we adopt the opportunistic multiuser scheduling proposed in [9]. In this policy, only one destination with the highest instantaneous end-to-end SNR out of K destinations is scheduled for transmission. This policy holds for a number of time-slotted downlink multiuser systems including Qualcomm's HDR [18] and high-speed downlink packet access [19]. In this case, if γ_s denotes the highest instantaneous end-to-end SNR of the scheduled destination (i.e., *strongest* destination), then the policy is formulated as

$$\gamma_s = \max_{1\leq k\leq K}\{\gamma_{\mathrm{eq},k}\} \tag{6}$$

where $\gamma_{\mathrm{eq},k}$ is defined in (3).

The scheduling algorithm is implemented at the relay, which is assumed to have knowledge of the relay–destination links of the K destinations that are scheduled for transmission. At the beginning of each scheduling period, the relay broadcasts a destination synchronization message (DSM), which contains the synchronization information and the pilot signaling transmission order. We assume that the destinations are perfectly synchronized to the relay in time. This is a common assumption in TDMA cellular networks [20], [21]. Each destination conveys pilot signals to the relay according to the transmission order received in the DSM. At the relay, channel estimation is conducted based on a pilot signal sequence sent by the K destinations. The relay identifies and selects the *strongest* destination and then feeds back the index of the *strongest* destination to the source. The destination selection is assumed to be done at discrete instants of time $t = nT$, where T is the selection period. This can be accommodated by introducing a short guard interval every selection period, during which the relay performs channel estimations for the relay–destination links and necessary comparisons for destination selection. To avoid the frequent selection operation between destinations, we assume that the relay updates the destination selection only during this guard interval before each data burst and that the length of the data burst is on the order of the channel coherence time.

III. Outage Probability Analysis

We characterize the performance of MRNs in terms of the outage probability. In particular, we provide a unified approach to derive the cdf of γ_s, under the assumption of two different gain constraints, i.e., CSI-based- and fixed-gain relaying, in Nakagami-m fading channel conditions. By relying on these statistical results, the outage probability is evaluated. The outage probability $P_{\rm out}$ is defined as the probability that γ_s drops below a predetermined specified SNR threshold $\gamma_{\rm th}$. Therefore, $P_{\rm out}$ is given by

$$P_{\rm out} = \Pr[\gamma_s < \gamma_{\rm th}] = F_{\gamma_s}(\gamma_{\rm th}) \tag{7}$$

where $F_{\gamma_s}(\gamma_{\rm th})$ is the cdf of γ_s, evaluated at $\gamma = \gamma_{\rm th}$. In the following, we evaluate exact closed-form expressions for $F_{\gamma_s}(\gamma_{\rm th})$ under the CSI-based-gain constraint $F_{\gamma_{s,\rm CSI}}(\gamma_{\rm th})$ and the fixed-gain constraint $F_{\gamma_{s,\rm Fix}}(\gamma_{\rm th})$.

A. CSI-Based-Gain MRN: General Results

In this subsection, we consider MRNs equipped with a CSI-based-gain relay. CSI-based-gain relaying adopts channel measurements to counteract the fading effects of the source–relay link. When the noise statistical information is not available at the relay, the scaling gain is set to $G^2 = 1/(|\alpha|^2 d_{\rm SR}^{-\eta} E_{\rm S})$ [5]. In this case, the instantaneous end-to-end SNR of the kth destination can be rewritten as

$$\gamma_{{\rm eq},k,\rm CSI} = \frac{\gamma_1 \gamma_{2,k}}{\gamma_1 + \gamma_{2,k}}. \tag{8}$$

Although systems with CSI-based-gain relaying are expected to outperform those with fixed-gain relaying, this improvement comes at the inevitable cost of increased power consumption, channel-information overhead, and circuitry footprint.

Theorem 1 (CDF of $\gamma_{s,\rm CSI}$ in CSI-Based-Gain MRNs): Under the assumption of unbalanced hops (i.e., $m_1 \neq m_2$ and $\overline{\gamma}_1 \neq \overline{\gamma}_2$) and the fact that the K relay–destination links undergo independent identically distributed (i.i.d.) fading channels, the cdf of the highest instantaneous end-to-end SNR $\gamma_{s,\rm CSI}$ of the *strongest* destination is derived as

$$\begin{aligned} F_{\gamma_{s,\rm CSI}}(\gamma) &= 1 + \frac{2m_1^{m_1}}{\overline{\gamma}_1^{m_1}\Gamma(m_1)} \sum_{j=0}^{m_1-1} \binom{m_1-1}{j} \\ &\quad \times \sum_{i=0}^{K-1} \binom{K}{i} (-1)^{K-i} \sum_{n_1=0}^{K-i} \sum_{n_2=0}^{n_1} \cdots \sum_{n_{m_2-1}=0}^{n_{m_2-2}} \frac{(K-i)!}{n_{m_2-1}!} \\ &\quad \times \prod_{t=1}^{m_2-1} \left[\frac{1}{(n_{t-1}-n_t)!(t!)^{n_t-n_{t+1}}} \right] \left(\frac{m_2}{\overline{\gamma}_2} \right)^{\phi} \\ &\quad \times \sum_{s=0}^{\phi} \binom{\phi}{s} \left(\frac{m_2 \overline{\gamma}_1 (K-i)}{m_1 \overline{\gamma}_2} \right)^{\frac{1}{2}(m_1-j-s)} \\ &\quad \times \gamma^{m_1+\phi} e^{-\left[\frac{m_1}{\overline{\gamma}_1} + \frac{m_2}{\overline{\gamma}_2}(K-i)\right]\gamma} \\ &\quad \times K_{m_1-j-s}\left(2\gamma \sqrt{\frac{m_1 m_2 (K-i)}{\overline{\gamma}_1 \overline{\gamma}_2}} \right) \end{aligned} \tag{9}$$

where $K_v(\cdot)$ denotes the vth-order modified Bessel function of the second kind, $\phi = \sum_{q=1}^{m_2-1} n_q = n_1 + n_2 + \cdots + n_{m_2-1}$, $n_0 = K - i$, and $n_{m_2} = 0$.

Proof: See Appendix A. ■

Consequently, the outage probability of MRNs with CSI-based-gain relaying is evaluated by substituting (9) into (7) as

$$P_{\rm out,CSI} = F_{\gamma_{s,\rm CSI}}(\gamma_{\rm th}). \tag{10}$$

This is a new exact outage probability expression that is valid for arbitrary integer values of the fading parameter m. It is worth noting that (10) can be calculated in closed form, involving only finite summations of exponentials, powers, and Bessel functions.

Corollary 1 (PDF of $\gamma_{s,\rm CSI}$ in CSI-Based-Gain MRNs): The pdf of $\gamma_{s,\rm CSI}$ is given by

$$\begin{aligned} f_{\gamma_{s,\rm CSI}}(\gamma) &= \frac{2m_1^{m_1}}{\overline{\gamma}_1^{m_1}\Gamma(m_1)} \sum_{j=0}^{m_1-1} \binom{m_1-1}{j} \\ &\quad \times \sum_{i=0}^{K-1} \binom{K}{i} (-1)^{K-i} \sum_{n_1=0}^{K-i} \sum_{n_2=0}^{n_1} \cdots \sum_{n_{m_2-1}=0}^{n_{m_2-2}} \frac{(K-i)!}{n_{m_2-1}!} \\ &\quad \times \prod_{t=1}^{m_2-1} \left[\frac{1}{(n_{t-1}-n_t)!(t!)^{n_t-n_{t+1}}} \right] \left(\frac{m_2}{\overline{\gamma}_2} \right)^{\phi} \\ &\quad \times \sum_{s=0}^{\phi} \binom{\phi}{s} \left(\frac{m_2 \overline{\gamma}_1 (K-i)}{m_1 \overline{\gamma}_2} \right)^{\frac{1}{2}(m_1-j-s)} \\ &\quad \times \gamma^{m_1+\phi} e^{-\left[\frac{m_1}{\overline{\gamma}_1} + \frac{m_2}{\overline{\gamma}_2}(K-i)\right]\gamma} \\ &\quad \times \left[\Theta_1 K_{m_1-j-s}(\Theta_2 \gamma) - \Theta_2 K_{m_1-j-s-1}(\Theta_2 \gamma) \right] \end{aligned} \tag{11}$$

where the following notations are introduced for compactness: $\Theta_1 = (\phi + j + s/\gamma) - [(m_1/\overline{\gamma}_1) + (m_2/\overline{\gamma}_2)(K-i)]$, and $\Theta_2 = 2\sqrt{(m_1 m_2 (K-i)/\overline{\gamma}_1\overline{\gamma}_2)}$.

Proof: The proof follows by differentiating (9) with respect to γ and using [17, eq. (8.486.12)] for the derivative of $K_v(\cdot)$. ■

B. CSI-Based-Gain MRN: Results for Special Cases

We now show two special cases for the $P_{\text{out,CSI}}$ of CSI-based-gain MRNs, as discussed below.

Special Case 1: For the single-user point-to-point dual-hop link in Nakagami-m fading, $P_{\text{out,CSI}}$ is found by setting $K = 1$ in (10) as

$$P_{\text{out,CSI}} = 1 - \frac{2}{\Gamma(m_1)} \sum_{j=0}^{m_1-1} \sum_{k=0}^{m_2-1} \sum_{s=0}^{k} \Xi[j,k,s] \gamma_{\text{th}}^{m_1+k} \times e^{-\left(\frac{m_1}{\overline{\gamma}_1} + \frac{m_2}{\overline{\gamma}_2}\right)\gamma_{\text{th}}} K_{m_1-j-s}\left(2\gamma_{\text{th}}\sqrt{\frac{m_1 m_2}{\overline{\gamma}_1\overline{\gamma}_2}}\right) \quad (12)$$

where

$$\Xi[j,k,s] = \frac{\binom{m_1-1}{j}\binom{k}{s}}{k!} \left(\frac{m_1}{\overline{\gamma}_1}\right)^{\frac{m_1+j+s}{2}} \left(\frac{m_2}{\overline{\gamma}_2}\right)^{\frac{m_1-j-s+2k}{2}}.$$

Note that a similar expression to (12) has been derived in [8, eq. (12)]. Moreover, when balanced hops are assumed (e.g., $m_1 = m_2$ and $\overline{\gamma}_1 = \overline{\gamma}_2$), the expression in (12) serves as an alternative representation to [22, eq. (18)], which was expressed in terms of the Meijer's G-function defined in [17, eq. (9.301)].

In addition, the pdf of $\gamma_{s,\text{CSI}}$ for the single-user scenario in Nakagami-m fading is extracted by setting $K = 1$ in (11) as

$$f_{\gamma_{s,\text{CSI}}}(\gamma) = -\frac{2}{\Gamma(m_1)} \sum_{j=0}^{m_1-1} \sum_{k=0}^{m_2-1} \sum_{s=0}^{k} \Xi[j,k,s] \times \gamma^{m_1+k} e^{-\left(\frac{m_1}{\overline{\gamma}_1} + \frac{m_2}{\overline{\gamma}_2}\right)\gamma} \times \left[\Theta_1' K_{m_1-j-s}(\Theta_2'\gamma) - \Theta_2' K_{m_1-j-s-1}(\Theta_2'\gamma)\right] \quad (13)$$

where $\Theta_1' = (j + k + s/\gamma) - ((m_1/\overline{\gamma}_1) + (m_2/\overline{\gamma}_2))$, $\Theta_2' = 2\sqrt{(m_1 m_2/\overline{\gamma}_1\overline{\gamma}_2)}$, and $\Xi[j,k,s]$ is defined in (12).

Special Case 2: For the multiuser point-to-multipoint dual-hop link in Rayleigh fading, $P_{\text{out,CSI}}$ is found by setting $m_1 = m_2 = 1$ in (10) as

$$P_{\text{out,CSI}} = 1 + \frac{2\gamma_{\text{th}}}{\sqrt{\overline{\gamma}_1\overline{\gamma}_2}} \sum_{i=0}^{K-1} \binom{K}{i} (-1)^{K-i} \sqrt{K-i} \times e^{-\left(\frac{1}{\overline{\gamma}_1} + \frac{K-i}{\overline{\gamma}_2}\right)\gamma_{\text{th}}} K_1\left(2\gamma_{\text{th}}\sqrt{\frac{K-i}{\overline{\gamma}_1\overline{\gamma}_2}}\right). \quad (14)$$

It is noted that (14) coincides with the previously published result in [16, eq. (5)]. This demonstrates the effectiveness and the validity of our result in (10).

Subsequently, the pdf of $\gamma_{s,\text{CSI}}$ for the multiuser scenario in Rayleigh fading is obtained by setting $m_1 = m_2 = 1$ in (11) as

$$f_{\gamma_{s,\text{CSI}}}(\gamma) = \frac{2\gamma}{\sqrt{\overline{\gamma}_1\overline{\gamma}_2}} \sum_{i=0}^{K-1} \binom{K}{i} (-1)^{K-i} \sqrt{K-i} \times e^{-\left(\frac{1}{\overline{\gamma}_1} + \frac{K-i}{\overline{\gamma}_2}\right)\gamma} \left[\Theta_1'' K_1(\Theta_2''\gamma) - \Theta_2'' K_0(\Theta_2''\gamma)\right] \quad (15)$$

where $\Theta_1'' = -((1/\overline{\gamma}_1) + (K - i/\overline{\gamma}_2))$, and $\Theta_2'' = 2\sqrt{(K - i/\overline{\gamma}_1\overline{\gamma}_2)}$. Note that the result in (15) can also be obtained by taking the derivative of (14) with respect to γ_{th} and setting $\gamma_{\text{th}} = \gamma$.

C. Fixed-Gain MRN: General Results

We now turn our attention to fixed-gain relaying, where the fading coefficients of the source–relay link are not known to the relay. As such, the relay introduces a fixed scaling gain of $G^2 = 1/(CN_0)$ to its received signal [6], where $C = 1 + \overline{\gamma}_1$. Therefore, the end-to-end instantaneous SNR of the kth destination is given by

$$\gamma_{\text{eq},k,\text{Fix}} = \frac{\gamma_1\gamma_{2,k}}{\gamma_{2,k} + C}. \quad (16)$$

From an implementation standpoint, the need for low complexity and/or processing power savings have motivated the study of fixed-gain relaying, although from a performance standpoint, fixed-gain relaying is not expected to perform as well as CSI-based-gain relaying.

Theorem 2 (CDF of $\gamma_{s,\text{Fix}}$ in Fixed-Gain MRNs): Assuming unbalanced hops and the fact that the K relay–destination links experience i.i.d. fading conditions, the cdf of the highest instantaneous end-to-end SNR $\gamma_{s,\text{Fix}}$ of the *strongest* destination is derived as

$$\begin{aligned} F_{\gamma_{s,\text{Fix}}}(\gamma) &= 1 + \frac{2m_1^{m_1}}{\overline{\gamma}_1^{m_1}\Gamma(m_1)} \sum_{j=0}^{m_1-1} \binom{m_1-1}{j} \times \sum_{i=0}^{K-1} \binom{K}{i} (-1)^{K-i} \sum_{n_1=0}^{K-i} \sum_{n_2=0}^{n_1} \cdots \sum_{n_{m_2-1}=0}^{n_{m_2-2}} \frac{(K-i)!}{n_{m_2-1}!} \\ &\times \prod_{t=1}^{m_2-1} \left[\frac{1}{(n_{t-1}-n_t)!(t!)^{n_t-n_{t+1}}}\right] \times \left(\frac{m_2 C}{\overline{\gamma}_2}\right)^{\frac{1}{2}(m_1-j+\phi)} \left(\frac{\overline{\gamma}_1(K-i)}{m_1}\right)^{\frac{1}{2}(m_1-j-\phi)} \\ &\times \gamma^{\frac{1}{2}(m_1+j+\phi)} e^{-\frac{m_1}{\overline{\gamma}_1}\gamma} \times K_{m_1-j-\phi}\left(2\sqrt{\frac{m_1 m_2 (K-i) C\gamma}{\overline{\gamma}_1\overline{\gamma}_2}}\right) \end{aligned} \quad (17)$$

where $K_v(\cdot)$ denotes the vth-order modified Bessel function of the second kind, $\phi = \sum_{q=1}^{m_2-1} n_q = n_1 + n_2 + \cdots + n_{m_2-1}$, $n_0 = K - i$, and $n_{m_2} = 0$.

Proof: See Appendix B. ■

Using (17), the outage probability of MRNs with fixed-gain relaying is calculated as

$$P_{\mathrm{out,Fix}} = F_{\gamma_{s,\mathrm{Fix}}}(\gamma_{\mathrm{th}}). \tag{18}$$

To the best of our knowledge, this result is new. It is observed from (18) that this explicit outage probability expression contains only standard functions and can therefore be easily and efficiently evaluated numerically with software packages such as MATLAB.

Corollary 2 (PDF of $\gamma_{s,\mathrm{Fix}}$ in Fixed-Gain MRNs): The pdf of $\gamma_{s,\mathrm{Fix}}$ is shown as

$$\begin{aligned} f_{\gamma_{s,\mathrm{Fix}}}(\gamma) &= \frac{2m_1^{m_1}}{\overline{\gamma}_1^{m_1}\Gamma(m_1)} \sum_{j=0}^{m_1-1} \binom{m_1-1}{j} \\ &\times \sum_{i=0}^{K-1} \binom{K}{i} (-1)^{K-i} \sum_{n_1=0}^{K-i} \sum_{n_2=0}^{n_1} \cdots \sum_{n_{m_2-1}=0}^{n_{m_2-2}} \frac{(K-i)!}{n_{m_2-1}!} \\ &\times \prod_{t=1}^{m_2-1} \left[\frac{1}{(n_{t-1}-n_t)!(t!)^{n_t-n_{t+1}}} \right] \\ &\times \left(\frac{m_2 C}{\overline{\gamma}_2} \right)^{\frac{1}{2}(m_1-j+\phi)} \left(\frac{\overline{\gamma}_1(K-i)}{m_1} \right)^{\frac{1}{2}(m_1-j-\phi)} \\ &\times \gamma^{\frac{1}{2}(m_1+j+\phi)} e^{-\frac{m_1}{\overline{\gamma}_1}\gamma} \\ &\times \left[\Delta_1 K_{m_1-j-\phi}(2\Delta_2\gamma) - \Delta_2 K_{m_1-j-\phi-1}(2\Delta_2\gamma) \right] \end{aligned} \tag{19}$$

where we introduce $\Delta_1 = (j+\phi/\gamma) - (m_1/\overline{\gamma}_1)$ and $\Delta_2 = \sqrt{(m_1 m_2 (K-i) C/\overline{\gamma}_1\overline{\gamma}_2\gamma)}$ for compactness.

Proof: The proof follows by differentiating (17) with respect to γ and using [17, eq. (8.486.12)] for the derivative of $K_v(\cdot)$. ■

D. Fixed-Gain MRN: Results for Special Cases

Next, we present two special cases for $P_{\mathrm{out,Fix}}$ of fixed-gain MRNs as follows.

Special Case 1: For the single-user point-to-point dual-hop link in Nakagami-m fading, $P_{\mathrm{out,Fix}}$ is found by setting $K=1$ in (18) as

$$\begin{aligned} P_{\mathrm{out,Fix}} = 1 - \frac{2}{\Gamma(m_1)} \sum_{j=0}^{m_1-1} \sum_{k=0}^{m_2-1} \Xi[j,k] \gamma_{\mathrm{th}}^{\frac{m_1+j+k}{2}} \\ \times e^{-\frac{m_1}{\overline{\gamma}_1}\gamma_{\mathrm{th}}} K_{m_1-j-k}\left(2\sqrt{\frac{m_1 m_2 C \gamma_{\mathrm{th}}}{\overline{\gamma}_1\overline{\gamma}_2}} \right) \end{aligned} \tag{20}$$

where

$$\Xi[j,k] = \frac{\binom{m_1-1}{j}}{k!} \left(\frac{m_1}{\overline{\gamma}_1} \right)^{\frac{m_1+j+k}{2}} \left(\frac{m_2 C}{\overline{\gamma}_2} \right)^{\frac{m_1-j+k}{2}}. \tag{21}$$

It is noted that (20) is equivalent to [8, eq. (18)]. Hence, our result in (18) stands for a generalization of the single-user dual-hop relay link.

Furthermore, the pdf of $\gamma_{s,\mathrm{Fix}}$ for the single-user point-to-point dual-hop link in Nakagami-m fading is found by setting $K=1$ in (19) as

$$\begin{aligned} f_{\gamma_{s,\mathrm{Fix}}}(\gamma) = -\frac{2}{\Gamma(m_1)} \sum_{j=0}^{m_1-1} \sum_{k=0}^{m_2-1} \Xi[j,k] \gamma^{\frac{m_1+j+k}{2}} e^{-\frac{m_1}{\overline{\gamma}_1}\gamma} \\ \times \left[\Delta_1' K_{m_1-j-\phi}(2\Delta_2'\gamma) - \Delta_2' K_{m_1-j-\phi-1}(2\Delta_2'\gamma) \right] \end{aligned} \tag{22}$$

where $\Delta_1' = \Delta_1 = (j+k/\gamma) - (m_1/\overline{\gamma}_1)$, $\Delta_2' = \sqrt{(m_1 m_2/\overline{\gamma}_1\overline{\gamma}_2\gamma)}$, and $\Xi[j,k]$ is defined in (20).

Special Case 2: For the multiuser point-to-multipoint dual-hop link in Rayleigh fading, we provide a new result for $P_{\mathrm{out,Fix}}$ by setting $m_1 = m_2 = 1$ in (18) as

$$\begin{aligned} P_{\mathrm{out,Fix}} = 1 + 2\sqrt{\frac{C\gamma_{\mathrm{th}}}{\overline{\gamma}_1\overline{\gamma}_2}} \sum_{i=0}^{K-1} \binom{K}{i} (-1)^{K-i} \sqrt{K-i} \\ \times e^{-\frac{\gamma_{\mathrm{th}}}{\overline{\gamma}_1}} K_1\left(2\sqrt{\frac{(K-i)C\gamma_{\mathrm{th}}}{\overline{\gamma}_1\overline{\gamma}_2}} \right). \end{aligned} \tag{23}$$

This special case has not previously been addressed.

Moreover, the pdf of $\gamma_{s,\mathrm{Fix}}$ for the multiuser point-to-multipoint dual-hop link in Rayleigh fading is found by setting $m_1 = m_2 = 1$ in (19) as

$$\begin{aligned} f_{\gamma_{s,\mathrm{Fix}}}(\gamma) = 2\sqrt{\frac{C}{\overline{\gamma}_1\overline{\gamma}_2}} \sum_{i=0}^{K-1} \binom{K}{i} (-1)^{K-i} \sqrt{K-i} e^{-\frac{\gamma}{\overline{\gamma}_1}} \\ \times \left[\Delta_1'' K_1(2\Delta_2''\gamma) - \Delta_2'' K_0(2\Delta_2''\gamma) \right] \end{aligned} \tag{24}$$

where $\Delta_1'' = -(\sqrt{\gamma}/\overline{\gamma}_1)$, and $\Delta_2'' = \sqrt{((K-i)C/\overline{\gamma}_1\overline{\gamma}_2)}$. Note that the result in (24) can be also obtained by taking the derivative of (23) with respect to γ_{th} and setting $\gamma_{\mathrm{th}} = \gamma$.

E. Achievable Diversity Order

We now focus on the achievable diversity order of MRNs to gain further insight into the impact of unbalanced hops on the system performance. Let

$$d = \lim_{\substack{\overline{\gamma}_1,\overline{\gamma}_2 \to \infty \\ \overline{\gamma}_2/\overline{\gamma}_1 \to \rho}} \frac{-\log P_{\mathrm{out}}}{\log \overline{\gamma}_1} \tag{25}$$

be the achievable diversity order, where ρ is a nonzero finite constant. Therefore, the achievable diversity order of MRNs is quantified by

$$d_{\mathrm{MRN}} = \min\{m_1, m_2 K\}. \tag{26}$$

Our result reveals that the diversity order achieved is jointly determined by the per-hop fading parameters and the number of destinations. From (26), we characterize the qualities of the first and second hops by m_1 and $m_2 K$, respectively, and refer to the lower quality hop as the weaker hop. It can be concluded from (26) that the system performance is limited by the weaker hop.

Furthermore, it is remarked that both CSI-based- and fixed-gain relaying achieve the same diversity order.

IV. Moment Analysis of γ_s

We now derive the moments of γ_s that can efficiently be applied to uncover other statistical measures. For instance, the average SNR and the AoF can be evaluated with the first-order and second-order moments. In addition, the moments of γ_s can be used to study higher order metrics such as the skewness and the kurtosis that characterize the distribution of γ_s.

By definition, the nth moment of γ_s is given by

$$\mathbf{E}\left[\gamma_s^n\right] = \int_0^\infty \gamma^n f_{\gamma_s}(\gamma) d\gamma. \tag{27}$$

Armed with the results for the pdf provided in Section III, we present exact closed-form expressions for the moments of γ_s as follows.

A. CSI-Based-Gain MRN

Theorem 3 (Moments of $\gamma_{s,\mathrm{CSI}}$ in CSI-Based-Gain MRNs): The nth moment of $\gamma_{s,\mathrm{CSI}}$ is given by

$$\begin{aligned}
&\mathbf{E}\left[\gamma_{s,\mathrm{CSI}}^n\right] \\
&= -\frac{n\sqrt{\pi}}{\Gamma(m_1)} \sum_{j=0}^{m_1-1} \binom{m_1-1}{j} \sum_{i=0}^{K-1} \binom{K}{i} (-1)^{K-i} \\
&\quad \times \sum_{n_1=0}^{K-i} \sum_{n_2=0}^{n_1} \cdots \sum_{n_{m_2-1}=0}^{n_{m_2-2}} \frac{(K-i)!}{n_{m_2-1}!} \\
&\quad \times \prod_{t=1}^{m_2-1} \left[\frac{1}{(n_{t-1}-n_t)!(t!)^{n_t-n_{t+1}}} \right] \\
&\quad \times \sum_{s=0}^{\phi} \binom{\phi}{s} \left(\frac{m_1}{\overline{\gamma}_1}\right)^{j+s} \left(\frac{m_2}{\overline{\gamma}_2}\right)^{\phi} 2^{-(n+2m_1-j+\phi-s)+1} \\
&\quad \times \left(\frac{m_1}{\overline{\gamma}_1} + \frac{m_2}{\overline{\gamma}_2}(K-i)\right)^{-n-j-\phi-s} \Gamma(n+j+\phi+s) \\
&\quad \times \frac{\Gamma(n+2m_1-j+\phi-s)}{\Gamma\left(n+m_1+\phi+\frac{1}{2}\right)} \\
&\quad \times {}_2F_1\left(\frac{n+j+\phi+s}{2}, \frac{n+j+\phi+s+1}{2}; \right. \\
&\qquad \left. n+m_1+\phi+\frac{1}{2}; 1-z_{\mathrm{CSI}}\right)
\end{aligned} \tag{28}$$

where ${}_2F_1(a,b;c;z)$ is Gauss's hypergeometric function, and

$$z_{\mathrm{CSI}} = \frac{4m_1m_2\overline{\gamma}_1\overline{\gamma}_2(K-i)}{\left(m_1\overline{\gamma}_2 + m_2\overline{\gamma}_1(K-i)\right)^2}.$$

Proof: This result is obtained by substituting (11) into (27) and solving the resultant integral with the aid of [23, eq. (2.16.6.3)]. ■

We now present two special cases for which the moments of $\gamma_{s,\mathrm{CSI}}$ in (28) reduce to a relatively simple form. In particular, we investigate the average SNR and the AoF of CSI-based-gain relaying.

1) Average SNR: By setting $n = 1$ in (28), the average SNR can be deduced in closed form as

$$\begin{aligned}
\overline{\gamma}_{s,\mathrm{CSI}} &= -\frac{\sqrt{\pi}}{\Gamma(m_1)} \Omega[m_1, m_2, K] \sum_{s=0}^{\phi} \binom{\phi}{s} \left(\frac{m_1}{\overline{\gamma}_1}\right)^{j+s} \left(\frac{m_2}{\overline{\gamma}_2}\right)^{\phi} \\
&\quad \times 2^{-(2m_1-j+\phi-s)} \left(\frac{m_1}{\overline{\gamma}_1} + \frac{m_2}{\overline{\gamma}_2}(K-i)\right)^{-\chi-1} \\
&\quad \times \frac{\Gamma(\chi+1)\Gamma(2m_1-j+\phi-s+1)}{\Gamma\left(m_1+\phi+\frac{3}{2}\right)} \\
&\quad \times {}_2F_1\left(\frac{\chi+1}{2}, \frac{\chi}{2}+1; m_1+\phi+\frac{3}{2}; 1-z_{\mathrm{CSI}}\right)
\end{aligned} \tag{29}$$

where $\chi = j + \phi + s$, and

$$\begin{aligned}
\Omega[m_1, m_2, K] &= \sum_{j=0}^{m_1-1} \binom{m_1-1}{j} \sum_{i=0}^{K-1} \binom{K}{i} (-1)^{K-i} \\
&\quad \times \sum_{n_1=0}^{K-i} \sum_{n_2=0}^{n_1} \cdots \sum_{n_{m_2-1}=0}^{n_{m_2-2}} \frac{(K-i)!}{n_{m_2-1}!} \\
&\quad \times \prod_{t=1}^{m_2-1} \left[\frac{1}{(n_{t-1}-n_t)!(t!)^{n_t-n_{t+1}}} \right].
\end{aligned} \tag{30}$$

2) AoF: The AoF characterizes the severity of the fading. With (28), the AoF can be expressed in a simple closed form as

$$A_{F,\mathrm{CSI}} = \frac{\mathbf{E}\left[\gamma_{s,\mathrm{CSI}}^2\right]}{\mathbf{E}[\gamma_{s,\mathrm{CSI}}]^2} - 1. \tag{31}$$

B. Fixed-Gain MRN

Theorem 4 (Moments of $\gamma_{s,\mathrm{Fix}}$ in Fixed-Gain MRNs): The nth moment of $\gamma_{s,\mathrm{Fix}}$ is given by

$$\begin{aligned}
\mathbf{E}\left[\gamma_{s,\mathrm{Fix}}^n\right] &= -\frac{n}{\Gamma(m_1)} \sum_{j=0}^{m_1-1} \binom{m_1-1}{j} \sum_{i=0}^{K-1} \binom{K}{i} (-1)^{K-i} \\
&\quad \times \sum_{n_1=0}^{K-i} \sum_{n_2=0}^{n_1} \cdots \sum_{n_{m_2-1}=0}^{n_{m_2-2}} \frac{(K-i)!}{n_{m_2-1}!} \\
&\quad \times \prod_{t=1}^{m_2-1} \left[\frac{1}{(n_{t-1}-n_t)!(t!)^{n_t-n_{t+1}}} \right] \\
&\quad \times \left(\frac{m_1}{\overline{\gamma}_1}\right)^{-n} \left(\frac{m_2 C}{\overline{\gamma}_2}\right)^{-n-j} (K-i)^{-n-j-\phi} \\
&\quad \times \Gamma(n+m_1)\Gamma(n+j+\phi) \\
&\quad \times {}_2F_0(n+m_1, n+j+\phi; ; -z_{\mathrm{Fix}})
\end{aligned} \tag{32}$$

where ${}_2F_0(a,b;;z)$ is the hypergeometric function, and

$$z_{\mathrm{Fix}} = \frac{\overline{\gamma}_2}{m_2(K-i)C}.$$

Proof: This result is obtained by substituting (19) into (27) and solving the resultant integral with the aid of [17, eq. (6.643.3)]. ■

Similar to the previous subsection, two special cases for the moments of $\gamma_{s,\mathrm{Fix}}$ in (32) are presented as follows.

1) Average SNR: By setting $n=1$ in (32), the average SNR can be calculated in closed form as

$$\begin{aligned}\overline{\gamma}_{s,\mathrm{Fix}} = &-\frac{1}{\Gamma(m_1)}\Omega[m_1,m_2,K]\left(\frac{m_1}{\overline{\gamma}_1}\right)^{-1}\left(\frac{m_2C}{\overline{\gamma}_2}\right)^{-j-1}\\ &\times (K-i)^{-j-\phi-1}\Gamma(m_1+1)\Gamma(j+\phi+1)\\ &\times {}_2F_0(m_1+1, j+\phi+1;\,;-z_{\mathrm{Fix}})\end{aligned} \tag{33}$$

where $\Omega[m_1,m_2,K]$ is given by (30).

2) AoF: Using (32), a simple closed-form expression for the AoF is given by

$$A_{F,\mathrm{Fix}} = \frac{\mathbf{E}\left[\gamma_{s,\mathrm{Fix}}^2\right]}{\mathbf{E}[\gamma_{s,\mathrm{Fix}}]^2} - 1. \tag{34}$$

V. Numerical Results and Discussion

In this section, we provide numerical examples to validate the theoretical analysis developed in the previous section. The performance of a network of K destinations is examined under various channel parameters. More specifically, we study the effect of a relay with unbalanced hops on the end-to-end outage probability by varying the per-hop average faded SNR (i.e., $\overline{\gamma}_1$ and $\overline{\gamma}_2$) and the per-hop fading parameter (i.e., m_1 and m_2).

Throughout this section, we assume that the source–relay–destination distance is normalized to unity such that $d_{\mathrm{SR}} + d_{\mathrm{RD}} = 1$. In addition, the variance of the fading coefficients is also normalized to unity with $\mathbf{E}[|\alpha|^2] = 1$ and $\mathbf{E}[|\beta_k|^2] = 1$. Moreover, equal average energies are assumed at the source and the relay with $E_{\mathrm{S}} = E_{\mathrm{R}}$, and the information symbols are modulated using binary phase-shift keying. As such, the first-hop and the second-hop average faded SNR is attenuated by $d_{\mathrm{SR}}^{-\eta}$ and $(1-d_{\mathrm{SR}})^{-\eta}$, respectively, and are jointly influenced by the path loss and the shadowing of obstacles. The path loss exponent η is an empirical constant that is often measured to describe the influence of the transmission medium. For outdoor propagation, η ranges between 2 and 5, depending upon the radio environment [24]. For instance, in free space, η is equal to 2, and when obstructions are present, η has a larger value. In this section, our results concentrate on the practical example of a highly shadowed urban area with $\eta = 4$. We use the "$\times$" symbol to mark points generated via Monte Carlo simulations. In each figure, the points precisely match the analytic curves, highlighting the accuracy of the analysis.

Fig. 2 depicts the cdf of the highest end-to-end SNR associated with the *strongest* destination in various Nakagami-m fading environments with $m_1 = m_2 = 1$ and $m_1 = m_2 = 2$. The curves are plotted across an SNR threshold range from 0 to 20 dB for $K = 1$ and 4, using (9) for CSI-based-gain relaying and (17) for fixed-gain relaying. Compared with a single-user scenario (i.e., $K = 1$), selecting the highest end-to-end SNR link in a multiuser scenario (i.e., $K = 4$) improves the system performance as expected. This performance improvement is further illustrated in Fig. 3, which presents the corresponding pdf shown in (11) and (19) for $m_1 = m_2 = 2$.

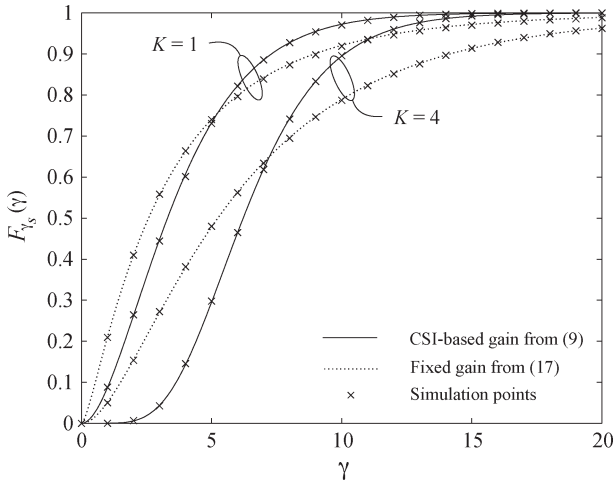

Fig. 2. CDF of $\gamma_{s,\mathrm{CSI}}$ and $\gamma_{s,\mathrm{Fix}}$ for $E_{\mathrm{S}}/N_0 = 0$ dB, $d_{\mathrm{SR}} = 0.3$, and $d_{\mathrm{RD}} = 0.7$ with $m_1 = m_2 = 2$.

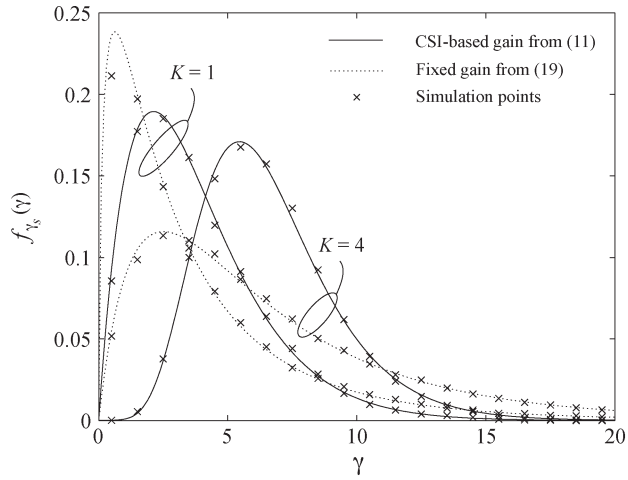

Fig. 3. PDF of $\gamma_{s,\mathrm{CSI}}$ and $\gamma_{s,\mathrm{Fix}}$ for $E_{\mathrm{S}}/N_0 = 0$ dB, $d_{\mathrm{SR}} = 0.3$, and $d_{\mathrm{RD}} = 0.7$ with $m_1 = m_2 = 2$.

A. Impact of the Relay Location

We investigate the impact of the relay location on the outage probability of MRNs by setting $d_{\mathrm{SR}} \neq d_{\mathrm{RD}}$ (thereby, $\overline{\gamma}_1 \neq \overline{\gamma}_2$) while keeping $m_1 = m_2$. This describes a wireless network where the relay is not placed halfway between the source and the destinations. Moreover, the relay and the destinations are both located within the same fading environment. In other words, the signals received at the relay and the destinations experience the same fading conditions. Here, we consider the fading conditions described as follows. When $m_1 = m_2 = 1$, it corresponds to the case where both the relay and the destinations are within a densely populated area with more severe

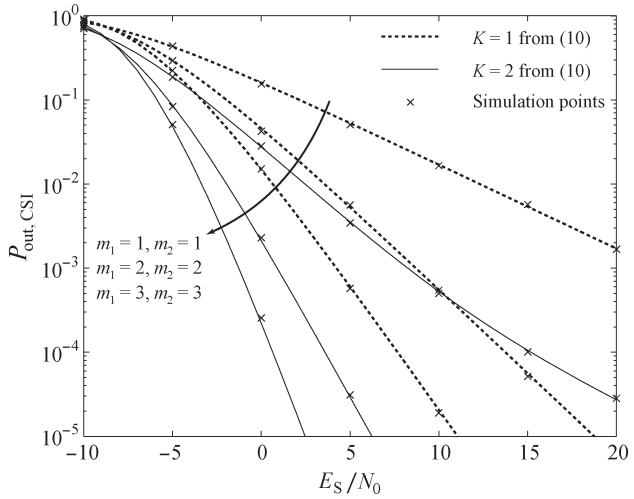

Fig. 4. Outage probability of CSI-based-gain MRNs for $\gamma_{\text{th}} = 0$ dB. *Case 1*: $d_{\text{SR}} = 0.3$, and $d_{\text{RD}} = 0.7$.

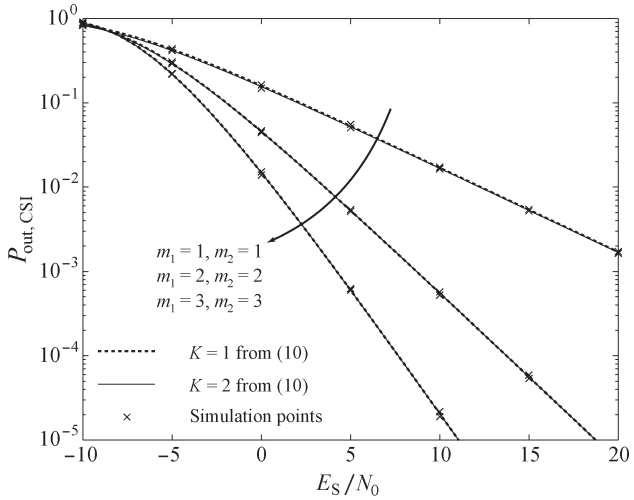

Fig. 6. Outage probability of CSI-based-gain MRNs for $\gamma_{\text{th}} = 0$ dB. *Case 2*: $d_{\text{SR}} = 0.7$, and $d_{\text{RD}} = 0.3$.

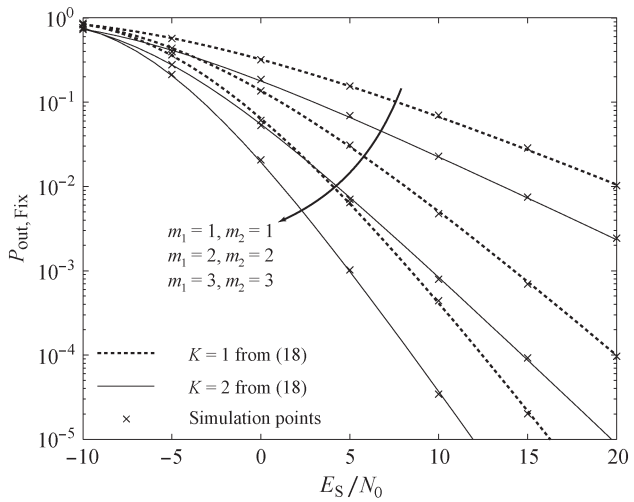

Fig. 5. Outage probability of fixed-gain MRNs for $\gamma_{\text{th}} = 0$ dB. *Case 1*: $d_{\text{SR}} = 0.3$, and $d_{\text{RD}} = 0.7$.

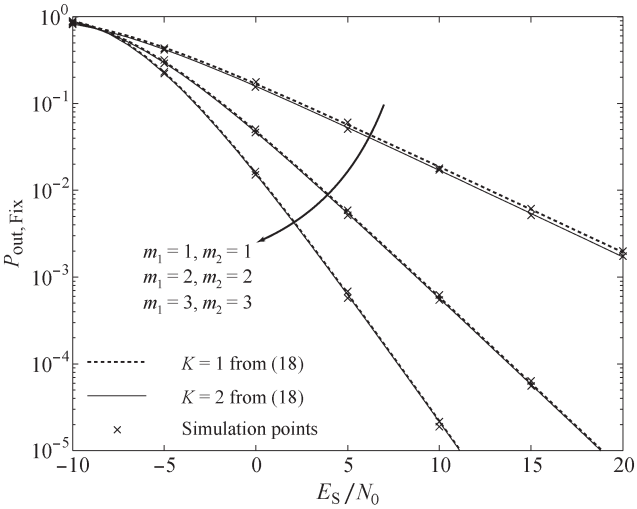

Fig. 7. Outage probability of fixed-gain MRNs for $\gamma_{\text{th}} = 0$ dB. *Case 2*: $d_{\text{SR}} = 0.7$, and $d_{\text{RD}} = 0.3$.

multipath fading; when $m_1 = m_2 = 2$, it corresponds to the case where both the relay and the destinations are within a moderate multipath environment with less scattering; and when $m_1 = m_2 = 3$, it corresponds to the case where the LOS component is dominant between the source and the relay and between the relay and the destinations.

We now show the outage probability of MRNs for $K = 1$ and $K = 2$, taking into account two cases based on d_{SR} and d_{RD}, as follows.

1) Case 1: The relay is located close to the source such that $d_{\text{SR}} = 0.3$ and $d_{\text{RD}} = 0.7$. Figs. 4 and 5 plot the outage probability against E_S/N_0 for CSI-based-gain relaying using (10) and fixed-gain relaying using (18), respectively. It is obvious that opportunistic scheduling offers a performance boost. For example, when $m_1 = m_2 = 3$ at the outage probability of 10^{-3} for CSI-based-gain relaying, it can be observed that $K = 2$ is superior by about 5.6 dB to the single-user scenario. On the other hand, for fixed-gain relaying, $K = 2$ outperforms the single-user scenario by about 3.5 dB.

2) Case 2: The relay is located close to the destinations such that $d_{\text{SR}} = 0.7$ and $d_{\text{RD}} = 0.3$. Figs. 6 and 7 plot the outage probability against E_S/N_0 for CSI-based-gain relaying using (10) and fixed-gain relaying using (18), respectively. For both CSI-based- and fixed-gain relaying, it can be observed that negligible gains are attained by opportunistic scheduling with $K = 2$, regardless of the fading parameter m. Moreover, it is shown that CSI-based-gain relaying yields a comparable outage probability with that of fixed-gain relaying. This is what we expect, since the presence of channel knowledge at the relay is less profitable as the channel quality of the first hop decreases, relative to that of the second hop.

These observations are not surprising since the performance of cooperative transmission is, in general, restricted by the weaker link. Specifically, when the relay sits closer to the source, as shown in *Case 1*, opportunistic scheduling will improve the channel quality of the second hop (i.e., weaker link), resulting in a considerable performance improvement. By contrast, when the relay sits closer to the destinations, as shown in *Case 2*, the first hop (i.e., weaker link) becomes the

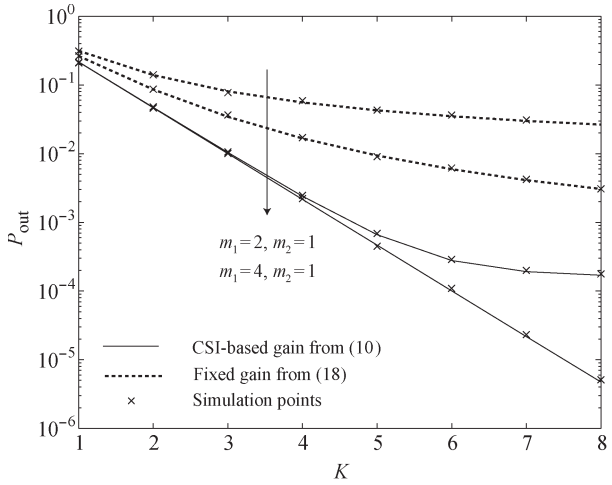

Fig. 8. Outage probability of MRNs as a function of K for CSI-based- and fixed-gain relaying with $E_S/N_0 = 0$ dB, $\gamma_{\text{th}} = 0$ dB, $d_{\text{SR}} = 0.3$, and $d_{\text{RD}} = 0.7$.

limiting factor. In this case, opportunistic scheduling will not improve the channel quality of the first hop, resulting in a less-noticeable performance improvement. The aforementioned observations address the key design question on where to place the relay to ensure the performance gains offered by opportunistic scheduling.

B. Impact of the Number of Destinations

We now examine the impact of the number of destinations on the outage probability of MRNs. Here, we model the fading conditions in downlink relay-assisted cellular applications. In this scenario, the source and the relay antennas are generally mounted on high towers to avoid obstructions, resulting in a dominant LOS transmission in the first hop with $m_1 \geq 2$. On the other hand, an NLOS transmission with $m_2 = 1$ is experienced in the second hop, where the signal transmitted from the relay to the destinations in urban areas is usually reflected from surrounding buildings, trees, and other obstructions.

Based on our previous observations, we choose to place the relay closer to the source such that $d_{\text{SR}} = 0.3$ and $d_{\text{RD}} = 0.7$. The outage probability of CSI-based- and fixed-gain MRNs is plotted versus K in Fig. 8. We observe that the outage probability decreases as K increases. This performance improvement is expected due to the fact that a higher K provides a higher variability in the instantaneous end-to-end SNRs among the different destinations. We also observe the impact of m_1 on the outage probability. Intuitively, the outage probability decreases as the fading severity decreases (i.e., m_1 increases). Furthermore, it is worth noticing that the multiuser advantage is more notable when the two hops are highly unbalanced (e.g., $m_1 = 4$ and $m_2 = 1$). This can be explained by the fact that the performance is profoundly influenced by the second hop, which is the weaker link. As a result, increasing K improves the link quality of the second hop, leading to a considerable reduction in outage probability. Finally, we find that the outage probability of CSI-based-gain relaying is lower than that of fixed-gain relaying. This is due to the fact that the CSI-based-gain relaying takes into account the channel conditions on the source–relay link.

VI. Conclusion

This paper has presented new closed-form expressions for the outage probability of MRNs employing a single AaF relay over generalized Nakagami-m fading channels. In particular, we have taken into consideration the realistic assumption of unbalanced hops and pointed out the effect of dissimilar per-hop fading parameters and/or dissimilar per-hop average faded SNRs. In doing so, we have derived exact expressions for the cdf and the pdf of the highest end-to-end SNR associated with the *strongest* destination in the cases of CSI-based- and fixed-gain relaying. For each case, we have also obtained closed-form expressions for the Rayleigh fading scenario and the single-user scenario as special cases. Moreover, closed-form expressions for the moments of the *strongest* end-to-end SNR have been presented, which were substantially applied to investigate important statistical parameters such as the average SNR and the AoF. Furthermore, we have studied the impact of the relay location on the system performance and developed a design criterion for the relay placement. Our proposed results provide a guideline to assist cellular infrastructure providers with the fundamental problem of relay placement to guarantee the multiuser advantage in practice.

Appendix A
Proof of Theorem 1

We now calculate the cdf of the highest instantaneous end-to-end SNR of the *strongest* destination for CSI-based-gain relaying $\gamma_{s,\text{CSI}}$. The cdf of the per-hop instantaneous faded SNRs γ_1 and $\gamma_{2,k}$, $k = 1, \ldots, K$ is given by (5). Under the assumption that the K relay–destination links undergo i.i.d. Nakagami fading, the cdf of $\gamma_{s,\text{CSI}}$ is calculated as

$$\begin{aligned} F_{\gamma_{s,\text{CSI}}}(\gamma) &= \int_0^\infty \Pr\left[\gamma_{s,\text{CSI}} < \gamma | \gamma_1\right] f_{\gamma_1}(\gamma_1) d\gamma_1 \\ &= \int_0^\infty \Pr\left[\frac{\gamma_1\gamma_{2,k}}{\gamma_1 + \gamma_{2,k}} < \gamma | \gamma_1\right]^K f_{\gamma_1}(\gamma_1) d\gamma_1 \\ &= \int_0^\gamma \Pr\left[\gamma_{2,k} > \frac{\gamma_1\gamma}{\gamma_1 - \gamma} | \gamma_1\right]^K f_{\gamma_1}(\gamma_1) d\gamma_1 \\ &\quad + \int_\gamma^\infty \Pr\left[\gamma_{2,k} < \frac{\gamma_1\gamma}{\gamma_1 - \gamma} | \gamma_1\right]^K f_{\gamma_1}(\gamma_1) d\gamma_1 \\ &= I_1 + I_2 \end{aligned} \tag{35}$$

where

$$I_1 = \int_0^\gamma f_{\gamma_1}(\gamma_1) d\gamma_1 = F_{\gamma_1}(\gamma) = 1 - \frac{\Gamma\left(m_1, \frac{m_1}{\overline{\gamma}_1}\gamma\right)}{\Gamma(m_1)} \tag{36}$$

$$\begin{aligned} I_2 &= \int_\gamma^\infty \left[F_{\gamma_{2,k}}\left(\frac{\gamma_1\gamma}{\gamma_1 - \gamma}\right)\right]^K f_{\gamma_1}(\gamma_1) d\gamma_1 \\ &= \int_\gamma^\infty \left[1 - \frac{\Gamma\left(m_2, \frac{m_2\gamma_1\gamma}{\overline{\gamma}_2(\gamma_1 - \gamma)}\right)}{\Gamma(m_2)}\right]^K \frac{m_1^{m_1}\gamma_1^{m_1-1}}{\overline{\gamma}_1^{m_1}\Gamma(m_1)} e^{-\frac{m_1}{\overline{\gamma}_1}\gamma_1} d\gamma_1. \end{aligned} \tag{37}$$

By using the binomial expansion in [17, eq. (1.111)], I_2 can be evaluated as

$$\begin{aligned} I_2 &= \int_{\gamma}^{\infty} \sum_{i=0}^{K} \binom{K}{i} (-1)^{K-i} \frac{\Gamma\left(m_2, \frac{m_2\gamma_1\gamma}{\overline{\gamma}_2(\gamma_1-\gamma)}\right)^{K-i}}{\Gamma(m_2)^{K-i}} \\ &\quad \times \frac{m_1^{m_1}\gamma_1^{m_1-1}}{\overline{\gamma}_1^{m_1}\Gamma(m_1)} e^{-\frac{m_1}{\overline{\gamma}_1}\gamma_1} d\gamma_1 \\ &= \frac{\Gamma\left(m_1, \frac{m_1}{\overline{\gamma}_1}\gamma\right)}{\Gamma(m_1)} + \frac{m_1^{m_1}}{\overline{\gamma}_1^{m_1}\Gamma(m_1)} \sum_{i=0}^{K-1} \frac{\binom{K}{i}(-1)^{K-i}}{\Gamma(m_2)^{K-i}} \\ &\quad \times \int_{\gamma}^{\infty} \Gamma\left(m_2, \frac{m_2\gamma_1\gamma}{\overline{\gamma}_2(\gamma_1-\gamma)}\right)^{K-i} \gamma_1^{m_1-1} e^{-\frac{m_1}{\overline{\gamma}_1}\gamma_1} d\gamma_1. \end{aligned} \tag{38}$$

Setting $\gamma_1 = x + \gamma$ in (38), I_2 can be reexpressed as

$$\begin{aligned} I_2 &= \frac{\Gamma\left(m_1, \frac{m_1}{\overline{\gamma}_1}\gamma\right)}{\Gamma(m_1)} + \frac{m_1^{m_1} e^{-\frac{m_1}{\overline{\gamma}_1}\gamma}}{\overline{\gamma}_1^{m_1}\Gamma(m_1)} \\ &\quad \times \sum_{j=0}^{m_1-1} \binom{m_1-1}{j} \gamma^j \sum_{i=0}^{K-1} \frac{\binom{K}{i}(-1)^{K-i}}{\Gamma(m_2)^{K-i}} I_3 \end{aligned} \tag{39}$$

where

$$I_3 = \int_0^{\infty} \Gamma\left(m_2, \frac{m_2\gamma}{\overline{\gamma}_2}\left(1+\frac{\gamma}{x}\right)\right)^{K-i} x^{m_1-1-j} e^{-\frac{m_1}{\overline{\gamma}_1}x} dx. \tag{40}$$

Applying [17, eq. (8.352.4)], the incomplete gamma function in the integrand of I_3 can be expanded as

$$\begin{aligned} \Gamma\left(m_2, \frac{m_2\gamma}{\overline{\gamma}_2}\left(1+\frac{\gamma}{x}\right)\right)^{K-i} &= (m_2-1)!^{K-i} \\ \times e^{-\frac{m_2\gamma}{\overline{\gamma}_2}(1+\frac{\gamma}{x})(K-i)} &\left[\sum_{p=0}^{m_2-1} \frac{\left(\frac{m_2\gamma}{\overline{\gamma}_2}\left(1+\frac{\gamma}{x}\right)\right)^p}{p!}\right]^{K-i}. \end{aligned} \tag{41}$$

To tackle the power of multinomial in (41), we invoke a useful formula given by [25, eq. (9)], yielding

$$\left[\sum_{p=0}^{m_2-1} \frac{\left(\frac{m_2\gamma}{\overline{\gamma}_2}\left(1+\frac{\gamma}{x}\right)\right)^p}{p!}\right]^{K-i} = \theta[m_2, K]\left(\frac{m_2\gamma}{\overline{\gamma}_2}\right)^{\phi} \sum_{s=0}^{\phi} \binom{\phi}{s} \frac{\gamma^s}{x^s} \tag{42}$$

where

$$\begin{aligned} \theta[m_2, K] &= \sum_{n_1=0}^{K-i} \sum_{n_2=0}^{n_1} \cdots \sum_{n_{m_2-1}=0}^{n_{m_2-2}} \frac{(K-i)!}{n_{m_2-1}!} \\ &\quad \times \prod_{t=1}^{m_2-1} \left[\frac{1}{(n_{t-1}-n_t)!(t!)^{n_t-n_{t+1}}}\right] \end{aligned} \tag{43}$$

with $n_0 = K-i$, $n_{m_2} = 0$, and $\phi = \sum_{q=1}^{m_2-1} n_q = n_1 + n_2 + \cdots + n_{m_2-1}$.

Therefore, I_3 can be solved by substituting (42) into (40) and applying [17, eq. (3.471.9)] as

$$\begin{aligned} I_3 &= (m_2-1)!^{K-i} e^{-\frac{m_2\gamma}{\overline{\gamma}_2}(K-i)} \theta[m_2, K] \left(\frac{m_2\gamma}{\overline{\gamma}_2}\right)^{\phi} \\ &\quad \times \sum_{s=0}^{\phi} \binom{\phi}{s} \gamma^s \int_0^{\infty} x^{m_1-1-j-s} e^{-\frac{m_2\gamma^2(K-i)}{\overline{\gamma}_2 x} - \frac{m_1 x}{\overline{\gamma}_1}} dx \\ &= 2(m_2-1)!^{K-i} e^{-\frac{m_2\gamma}{\overline{\gamma}_2}(K-i)} \theta[m_2, K] \left(\frac{m_2\gamma}{\overline{\gamma}_2}\right)^{\phi} \\ &\quad \times \sum_{s=0}^{\phi} \binom{\phi}{s} \gamma^s \left(\frac{m_2\gamma^2\overline{\gamma}_1(K-i)}{m_1\overline{\gamma}_2}\right)^{\frac{1}{2}(m_1-j-s)} \\ &\quad \times K_{m_1-j-s}\left(2\gamma\sqrt{\frac{m_1 m_2(K-i)}{\overline{\gamma}_1\overline{\gamma}_2}}\right). \end{aligned} \tag{44}$$

Thus, I_2 is evaluated by inserting (44) into (39). Finally, we obtain $F_{\gamma_{s,\mathrm{CSI}}}(\gamma)$ by using (36) together with (39).

Appendix B
Proof of Theorem 2

For fixed-gain relaying, following a similar procedure described in Appendix A, the cdf of $\gamma_{s,\mathrm{Fix}}$ shown in (17) is given by

$$\begin{aligned} F_{\gamma_{s,\mathrm{Fix}}}(\gamma) &= \int_0^{\infty} \Pr[\gamma_{s,\mathrm{Fix}} < \gamma|\gamma_1] f_{\gamma_1}(\gamma_1) d\gamma_1 \\ &= \int_0^{\infty} \Pr\left[\frac{\gamma_1\gamma_{2,k}}{\gamma_{2,k}+C} < \gamma|\gamma_1\right]^K f_{\gamma_1}(\gamma_1) d\gamma_1 \\ &= \int_0^{\gamma} \Pr\left[\gamma_{2,k} > \frac{\gamma C}{\gamma_1-\gamma}|\gamma_1\right]^K f_{\gamma_1}(\gamma_1) d\gamma_1 \\ &\quad + \int_{\gamma}^{\infty} \Pr\left[\gamma_{2,k} < \frac{\gamma C}{\gamma_1-\gamma}|\gamma_1\right]^K f_{\gamma_1}(\gamma_1) d\gamma_1 \\ &= I_1 + I_2. \end{aligned} \tag{45}$$

The integral I_1 in (45) is identical to that shown in (36). Similarly, the integral I_2 can be derived by using binomial expansion as follows:

$$\begin{aligned} I_2 &= \int_{\gamma}^{\infty} \left[1 - \frac{\Gamma\left(m_2, \frac{m_2 C\gamma}{\overline{\gamma}_2(\gamma_1-\gamma)}\right)}{\Gamma(m_2)}\right]^K \frac{m_1^{m_1}\gamma_1^{m_1-1}}{\overline{\gamma}_1^{m_1}\Gamma(m_1)} e^{-\frac{m_1}{\overline{\gamma}_1}\gamma} d\gamma_1 \\ &= \frac{\Gamma\left(m_1, \frac{m_1}{\overline{\gamma}_1}\gamma\right)}{\Gamma(m_1)} + \frac{m_1^{m_1}}{\overline{\gamma}_1^{m_1}\Gamma(m_1)} \sum_{i=0}^{K-1} \frac{\binom{K}{i}(-1)^{K-i}}{\Gamma(m_2)^{K-i}} \\ &\quad \times \int_{\gamma}^{\infty} \Gamma\left(m_2, \frac{m_2 C\gamma}{\overline{\gamma}_2(\gamma_1-\gamma)}\right)^{K-i} \gamma_1^{m_1-1} e^{-\frac{m_1}{\overline{\gamma}_1}\gamma_1} d\gamma_1. \end{aligned} \tag{46}$$

Setting $\gamma_1 = x + \gamma$ in (46) and using binomial expansion, I_2 can be reexpressed as

$$I_2 = \frac{\Gamma\left(m_1, \frac{m_1}{\overline{\gamma}_1}\gamma\right)}{\Gamma(m_1)} + \frac{m_1^{m_1} e^{-\frac{m_1}{\overline{\gamma}_1}\gamma}}{\overline{\gamma}_1^{m_1}\Gamma(m_1)} \times \sum_{j=0}^{m_1-1} \binom{m_1-1}{j} \gamma^j \sum_{i=0}^{K-1} \frac{\binom{K}{i}(-1)^{K-i}}{\Gamma(m_2)^{K-i}} I_3 \quad (47)$$

where

$$I_3 = \int_0^\infty \Gamma\left(m_2, \frac{m_2 C\gamma}{\overline{\gamma}_2 x}\right)^{K-i} x^{m_1-1-j} e^{-\frac{m_1}{\overline{\gamma}_1}x} dx. \quad (48)$$

Using [17, eq. (8.352.4)], [25, eq. (9)], and [17, eq. (3.471.9)], I_3 can be calculated as

$$I_3 = (m_2-1)!^{K-i}\theta[m_2, K]\left(\frac{m_2 C\gamma}{\overline{\gamma}_2}\right)^{\phi} \times \left(\frac{m_2\overline{\gamma}_1(K-i)C\gamma}{m_1\overline{\gamma}_2}\right)^{\frac{1}{2}(m_1-j-\phi)} \times K_{m_1-j-\phi}\left(2\sqrt{\frac{m_1 m_2(K-i)C\gamma}{\overline{\gamma}_1\overline{\gamma}_2}}\right) \quad (49)$$

where $\theta[m_2, K]$ is given by (43). As a result, the cdf of $\gamma_{s,\mathrm{Fix}}$ is found by substituting (36) and (47) with I_3 given in (49) into (45), resulting in (17).

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REFERENCES

[1] R. Pabst, B. H. Walke, D. C. Schultz, D. C. Herhold, H. Yanikomeroglu, S. Mukherjee, H. Viswanathan, M. Lott, W. Zirwas, M. Dohler, H. Aghvami, D. D. Falconer, and G. P. Fettweis, "Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Commun. Mag.*, vol. 42, no. 9, pp. 80–89, Sep. 2004.

[2] D. Soldani and S. Dixit, "Wireless relays for broadband access," *IEEE Commun. Mag.*, vol. 46, no. 3, pp. 58–66, Mar. 2008.

[3] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[4] M. O. Hasna and M.-S. Alouini, "Outage probability of multihop transmission over Nakagami fading channels," *IEEE Commun. Lett.*, vol. 7, no. 5, pp. 216–218, May 2003.

[5] M. O. Hasna and M.-S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 6, pp. 1126–1131, Nov. 2003.

[6] M. O. Hasna and M.-S. Alouini, "A performance study of dual-hop transmissions with fixed gain relays," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1963–1968, Nov. 2004.

[7] G. K. Karagiannidis, T. A. Tsiftsis, and R. K. Mallik, "Bounds for multihop relayed communications in Nakagami-m fading," *IEEE Trans. Commun.*, vol. 54, no. 1, pp. 18–22, Jan. 2006.

[8] T. A. Tsiftsis, G. K. Karagiannidis, P. T. Mathiopoulos, and S. A. Kotsopoulos, "Nonregenerative dual-hop cooperative links with selection diversity," *EURASIP J. Wireless Commun. Netw.*, vol. 2006, no. 2, p. 34, Apr. 2006, Article ID 17862.

[9] D. N. C. Tse, "Optimal power allocation over parallel Gaussian broadcast channels," in *Proc. IEEE ISIT*, Ulm, Germany, Jun. 1997, p. 27.

[10] C.-J. Chen and L.-C. Wang, "A unified capacity analysis for wireless systems with joint multiuser scheduling and antenna diversity in Nakagami fading channels," *IEEE Trans. Commun.*, vol. 54, no. 3, pp. 469–478, Mar. 2006.

[11] D. Piazza and L. B. Milstein, "Analysis of multiuser diversity in time-varying channels," *IEEE Trans. Wireless Commun.*, vol. 6, no. 12, pp. 4412–4419, Dec. 2007.

[12] L. Erwu, W. Dongyao, L. Jimin, S. Gang, and J. Shan, "Performance evaluation of bandwidth allocation in 802.16j mobile multi-hop relay networks," in *Proc. IEEE 65th VTC*, Dublin, Ireland, Apr. 2007, pp. 939–943.

[13] W. Mohr, "The WINNER (wireless world initiative new radio) project—Development of a radio interface for systems beyond 3G," in *Proc. IEEE 16th Int. Symp. PIMRC*, Berlin, Germany, Sep. 2005, pp. 857–862.

[14] M. S. Kang, B. C. Jung, and D. K. Sung, "Effect of cooperative and selection relaying schemes on multiuser diversity in downlink cellular systems with relays," *J. Commun. Netw.*, vol. 10, no. 2, pp. 175–185, Jun. 2008.

[15] H.-J. Joung and C. Mun, "Capacity of multiuser diversity with cooperative relaying in wireless networks," *IEEE Commun. Lett.*, vol. 12, no. 10, pp. 752–754, Oct. 2008.

[16] J.-B. Kim and D. Kim, "Comparison of two SNR-based feedback schemes in multiuser dual-hop amplify-and-forward relaying networks," *IEEE Commun. Lett.*, vol. 12, no. 8, pp. 557–559, Aug. 2008.

[17] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products,* 7th ed. San Diego, CA: Academic, 2007.

[18] P. Bender, P. Black, M. Grob, R. Padovani, N. Sindhushyana, and A. Viterbi, "CDMA/HDR: A bandwidth efficient high speed wireless data service for nomadic users," *IEEE Commun. Mag.*, vol. 38, no. 7, pp. 70–77, Jul. 2000.

[19] S. Parkvall, E. Dahlman, P. Frenger, P. Beming, and M. Persson, "The high speed packet data evolution of WCDMA," in *Proc. IEEE 12th Int. Symp. PIMRC*, San Diego, CA, Oct. 2001, pp. G-27–G-31.

[20] T. Tang and R. W. Heath, "Opportunistic feedback for downlink multiuser diversity," *IEEE Commun. Lett.*, vol. 9, no. 10, pp. 948–950, Oct. 2005.

[21] T. Tang, R. W. Heath, S. Cho, and S. Yun, "Opportunistic feedback for multiuser MIMO systems with linear receivers," *IEEE Trans. Commun.*, vol. 55, no. 5, pp. 1020–1032, May 2007.

[22] M. O. Hasna and M.-S. Alouini, "Harmonic mean and end-to-end performance of transmission systems with relays," *IEEE Trans. Commun.*, vol. 52, no. 1, pp. 130–135, Jan. 2004.

[23] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Integrals and Series*. New York: Gordon and Breach, 1986.

[24] X. Zhao, J. Kivinen, P. Vainikainen, and K. Skog, "Propagation characteristics for wideband outdoor mobile communications at 5.3 GHz," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 3, pp. 507–514, Apr. 2002.

[25] S. Choi and Y.-C. Ko, "Performance of selection MIMO systems with generalized selection criterion over Nakagami-m fading channels," *IEICE Trans. Commun.*, vol. E89-B, no. 12, pp. 3467–3470, Dec. 2006.

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