On the SER of Fixed Gain Amplify-and-Forward Relaying with Beamforming in Nakagami-m Fading

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***Abstract*—We derive the exact and asymptotic symbol error rate (SER) of multiple-input multiple-output (MIMO) relaying with a fixed gain amplify-and-forward (AaF) relay in Nakagami-m fading channels. We demonstrate that both fixed and variable gain relaying achieve the same diversity, which is jointly determined by the number of antennas and the per-hop fading parameters. We further quantify the performance gap between fixed and variable gain relaying in terms of their array gains. We prove that this gap diminishes as the first and the second hops exhibit highly asymmetric channel qualities.**

***Index Terms*—Multiple-input multiple-output, wireless relaying, beamforming, Nakagami-m fading.**

I. Introduction

By deploying multiple co-located antennas in wireless relay networks, multiple-input multiple-output (MIMO) relaying has attracted considerable interests in the recent years due to its ability to improving the transmission reliability [1, 2]. To examine the impacts of multiple antennas with beamforming in MIMO relaying, [3] presented the exact and asymptotic outage probability and the symbol error rate (SER) for variable gain relaying in Rayleigh fading channels. These results were extended to Nakagami-m fading channels in [4]. Nevertheless, the implementation of variable gain relaying implies a high complexity due to the requirement of perfect channel knowledge at the relay. Against this background, fixed gain relaying is applied to alleviate this requirement and thus becoming attractive from a cost-conscious perspective. Focusing on fixed gain MIMO relaying, [5] and [6] provided the outage probability and some useful statistics for Rayleigh and Nakagami-m fading channels, respectively.

In this letter, we derive new closed-form expressions for the exact and asymptotic SER of fixed gain MIMO relaying in Nakagami-m fading channels, which were not addressed in [6]. Specifically, beamforming is performed at the source with N_1 antennas and the destination with N_2 antennas. An essential conclusion is reached that both fixed and variable gain relaying achieve the same diversity order, which is equal to the minimum of $N_1 m_1$ and $N_2 m_2$ (where m_1 and m_2 denote the fading parameters of the first and second hops, respectively). The performance gap between fixed and variable gain relaying is further quantified in terms of their array gains. We demonstrate that both fixed and variable gain relaying asymptotically achieve the same SER when $N_1 m_1$ and $N_2 m_2$ are highly unbalanced.

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II. Fixed Gain MIMO Relaying

In the system under consideration, the communication between a multiple-antenna source and a multiple-antenna destination is facilitated by a single-antenna amplify-and-forward (AaF) relay. The source and the destination are equipped with N_1 and N_2 antennas, respectively. The source has no direct link with the destination, and the transmission is performed only via the relay in two successive time slots. In the first time slot, the source transmits the signal to the relay using maximum-ratio transmission (MRT) [7], whereas in the second time slot, the relay retransmits a scaled version of its received signal to the destination. The received signal at the destination is combined as per rules of maximum-ratio combining (MRC).

When a fixed gain is applied at the relay, the end-to-end instantaneous signal-to-noise ratio (SNR) at the destination is given by $\gamma_{\mathrm{D}} = \gamma_1 \gamma_2 / \gamma_2 + C$, where C is a constant, $\gamma_1 = ||\mathbf{h}||_F^2 E_{\mathrm{S}} / \mathcal{N}_0$ and $\gamma_2 = ||\mathbf{g}||_F^2 E_{\mathrm{R}} / \mathcal{N}_0$ are the instantaneous SNRs of the first and second hops, respectively, where $||\cdot||_F$ is the Frobenius norm. We denote $\mathbf{h}$ as the $N_1 \times 1$ channel vector of the first hop, $\mathbf{g}$ as the $N_2 \times 1$ channel vector of the second hop, E_{S} and E_{R} as the average energy available at the source and the relay, respectively, and $\mathcal{N}_0$ as the variance of the additive white Gaussian noises (AWGN) at the relay and each destination antenna. The average SNRs per antenna of the first and second hops are $\overline{\gamma}_1 = \mathbf{E}[\gamma_1]/N_1$ and $\overline{\gamma}_2 = \mathbf{E}[\gamma_2]$, respectively, where $\mathbf{E}[\cdot]$ denotes the expectation. The dual-hop transmission is assumed to experience independent but not necessarily identically distributed (i.n.d.) Nakagami-m fading with $m_1 \neq m_2$, where m_1 and m_2 denote the fading parameters of the first and second hops, respectively.

III. SER Analysis

A. Exact SER

We first present a new closed-form expression for the exact SER. We see in [6] that the exact SER can be evaluated based on the moment generating function (MGF) of γ_{D}. However, the intractability arises in seeking a closed-form solution to the finite-range integration of the MGF expression in [6] which involves the Whittaker function [9, eq. (9.220.4)]. As such, we invoke the cumulative distribution function (CDF) approach described in [8] and derive the exact SER in terms of the

CDF of $\gamma_{\rm D}$, $F_{\gamma_{\rm D}}(\gamma)$, as

$$P_s = \frac{a}{2}\sqrt{\frac{b}{\pi}}\int_0^\infty \gamma^{-\frac{1}{2}} e^{-b\gamma} F_{\gamma_{\rm D}}(\gamma) d\gamma. \tag{1}$$

This approach embraces all general modulation schemes that have an SER expression of the form $P_s = \mathbf{E}\left[a\mathcal{Q}\left(\sqrt{2b\gamma}\right)\right]$, where $\mathcal{Q}(\cdot)$ denotes the Gaussian-Q function and a and b are modulation specific constants.

Substituting $F_{\gamma_{\rm D}}(\gamma)$ from [6, eq. (10)] into (1), and applying [10, eq. (2.16.6.3)], we derive the exact SER as

$$P_s = \frac{a}{2} - \frac{a}{2}\sqrt{\frac{b}{\pi}}\sum_{n=0}^{N_1m_1-1}\sum_{k=0}^{n}\binom{n}{k}\left(\frac{m_2C}{\overline{\gamma}_2}\right)^{k-n-\frac{1}{2}}\left(\frac{m_1}{\overline{\gamma}_1}\right)^{-\frac{1}{2}} \times \frac{\Gamma(\sigma)\Gamma\left(n+\frac{1}{2}\right)}{n!\Gamma(N_2m_2)} {}_2F_0\left(\sigma, n+\frac{1}{2};\ ; -\frac{m_1\overline{\gamma}_2 + b\overline{\gamma}_1\overline{\gamma}_2}{m_1m_2C}\right), \tag{2}$$

where $\sigma = N_2m_2+n-k+\frac{1}{2}$, $\Gamma(x)$ is the gamma function, and ${}_2F_0(a,b;\ ;z)$ is the hypergeometric function. The modulation formats to which (2) apply include binary phase-shift keying (BPSK) ($a=1$, $b=1$), quadrature phase-shift keying (QPSK) ($a=1$, $b=0.5$), and M-ary pulse amplitude modulation (M-PAM) ($a = 2(M-1)/M$, $b = 3/(M^2-1)$).

B. Asymptotic SER

We now derive the asymptotic SER to determine the diversity order and the array gain, which enable us to explicitly examine the joint effects of beamforming and per-hop fading severities on the performance. Assuming that only the statistics of $\mathbf{h}$ is known at the relay, we define $C = 1 + \mathbf{E}\left[\|\mathbf{h}\|_F^2 E_{\rm S}/\mathcal{N}_0\right]$. By substituting $\overline{\gamma}_2 = \kappa\overline{\gamma}_1$ and $\gamma = \lambda\overline{\gamma}_1$ into $F_{\gamma_{\rm D}}(\gamma)$ from [6, eq. (10)], where κ and λ are positive real numbers, and using the Taylor series representation of the exponential and Bessel functions, we rewrite $F_{\gamma_{\rm D}}(\gamma)$ as

$$F_{\gamma_{\rm D}}(\gamma) = F(\lambda) = A(\lambda) - B(\lambda), \tag{3}$$

where

$$A(\lambda) = 1 - \Upsilon\sum_{q=0}^{N_2m_2-k-1}\frac{(-1)^q(N_2m_2-k-q-1)!}{\Gamma(N_2m_2)\, q!\kappa^{k+q}} \times m_1^{n+q} m_2^{k+q} N_1^{k+q}\lambda^{n+p+q}, \tag{4}$$

and

$$B(\lambda) = \Upsilon\sum_{q=0}^{\infty}\frac{(-1)^{N_2m_2-k+1} m_1^{N_2m_2+n-k+q} m_2^{N_2m_2+q}}{\Gamma(N_2m_2)\, q!(N_2m_2-k+q)!\kappa^{N_2m_2+q}} \times N_1{}^{N_2m_2+q}\lambda^{N_2m_2+n-k+p+q}\Omega, \tag{5}$$

with $\Upsilon = \sum_{n=0}^{N_1m_1-1}\sum_{k=0}^{n}\sum_{p=0}^{\infty}\binom{n}{k}(-m_1)^p/n!p!$ and $\Omega = \ln(m_1m_2N_1\lambda/\kappa) - \psi(q+1) - \psi(N_2m_2-k+q+1)$. We then find the first order expansion of $F(\lambda)$ around $\lambda = 0$ as

$$F(\lambda) = \begin{cases} \Phi_1\lambda^{N_1m_1} + o\left(\lambda^{N_1m_1}\right) & , \quad N_1m_1 < N_2m_2 \\ \Phi_2\lambda^{N_2m_2} + o\left(\lambda^{N_2m_2}\right) & , \quad N_1m_1 > N_2m_2 \\ \Phi_3\lambda^{N_1m_1} + o\left(\lambda^{N_1m_1}\right) & , \quad N_1m_1 = N_2m_2 , \end{cases} \tag{6}$$

where

$$\Phi_1 = \frac{m_1^{N_1m_1}}{\Gamma(N_2m_2)}\sum_{n=0}^{N_1m_1}\frac{(N_2m_2-n-1)!m_2^n}{n!(N_1m_1-n)!}\left(\frac{N_1}{\kappa}\right)^n, \tag{7}$$

$$\Phi_2 = \frac{(m_1m_2)^{N_2m_2}(N_1m_1-N_2m_2-1)!}{(N_2m_2)!(N_1m_1-1)!}\left(\frac{N_1}{\kappa}\right)^{N_2m_2}, \tag{8}$$

and

$$\Phi_3 = \sum_{n=0}^{N_1m_1-1}\frac{m_1^{N_1m_1}m_2^n}{\Gamma(N_1m_1)n!(N_1m_1-n)}\left(\frac{N_1}{\kappa}\right)^n + \frac{(m_1m_2)^{N_1m_1}}{((N_1m_1)!)^2}\left(\frac{N_1}{\kappa}\right)^{N_1m_1}\Theta, \tag{9}$$

with $\Theta = 1 - N_1m_1(\ln(N_1m_1m_2\lambda/\kappa) - 2\psi(1))$. Substituting $\lambda = \gamma/\overline{\gamma}_1$ into (6) yields the first order expansion of $F_{\gamma_{\rm D}}(\gamma)$. Using this expansion and after some algebraic manipulations, the asymptotic SER can be derived as

$$P_s^\infty = (G_a\overline{\gamma}_1)^{-G_d} + o\left(\overline{\gamma}_1^{-G_d}\right), \tag{10}$$

where the diversity order is

$$G_d = \min\{N_1m_1, N_2m_2\}, \tag{11}$$

and the array gain is

$$G_a = \begin{cases} b\left(\frac{a\Gamma\left(N_1m_1+\frac{1}{2}\right)}{2\sqrt{\pi}}\Phi_1\right)^{-\frac{1}{N_1m_1}} & , \quad N_1m_1 < N_2m_2 \\ b\left(\frac{a\Gamma\left(N_2m_2+\frac{1}{2}\right)}{2\sqrt{\pi}}\Phi_2\right)^{-\frac{1}{N_2m_2}} & , \quad N_1m_1 > N_2m_2 \\ b\left(\frac{a\Gamma\left(N_1m_1+\frac{1}{2}\right)}{2\sqrt{\pi}}\Phi_3^*\right)^{-\frac{1}{N_1m_1}} & , \quad N_1m_1 = N_2m_2 , \end{cases} \tag{12}$$

with Φ_1 and Φ_2 are given by (7) and (8), respectively, and

$$\Phi_3^* = \sum_{n=0}^{N_1m_1-1}\frac{m_1^{N_1m_1}m_2^n}{\Gamma(N_1m_1)n!(N_1m_1-n)}\left(\frac{N_1}{\kappa}\right)^n + \frac{(m_1m_2)^{N_1m_1}}{((N_1m_1)!)^2}\left(\frac{N_1}{\kappa}\right)^{N_1m_1}\Theta^*, \tag{13}$$

with $\Theta^* = 1 - N_1m_1(\ln(N_1m_1m_2/\kappa\overline{\gamma}_1) - 2\psi(1))$. Based on our results, the qualities of the first and second hops can be characterized by N_1m_1 and N_2m_2, respectively. We refer to the lower quality hop as the weaker hop. From (11) we conclude that the diversity order is entirely determined by the weaker hop.

C. Fixed versus Variable Gain Relaying

Next, we compare the asymptotic SER of fixed gain relaying in (10) with that of variable gain relaying in [4]. By contrasting (11) with [4, eq. (7)], we conclude that both fixed and variable gain relaying attain the same diversity order. We then turn our attention to their respective array gains. Specifically, we present a ratio of the array gains, $G_{a,\rm V}/G_{a,\rm F}$, as

$$\frac{G_{a,\rm V}}{G_{a,\rm F}} = \begin{cases} \left(\frac{(N_1m_1)!\Phi_1}{m_1^{N_1m_1}}\right)^{\frac{1}{N_1m_1}} , & N_1m_1 < N_2m_2 \\ \left(\frac{(N_2m_2)!\kappa^{N_2m_2}\Phi_2}{m_2^{N_2m_2}}\right)^{\frac{1}{N_2m_2}} , & N_1m_1 > N_2m_2 \\ \left(\frac{\kappa^{N_1m_1}\Phi_3^*}{(m_1\kappa)^{N_1m_1} + m_2^{N_1m_1}}\right)^{\frac{1}{N_1m_1}} , & N_1m_1 = N_2m_2 , \end{cases} \tag{14}$$

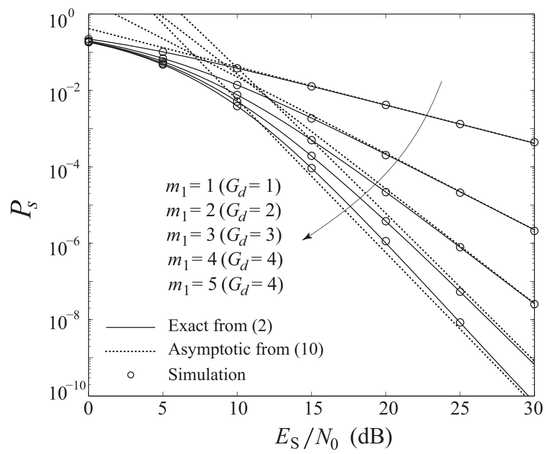

Fig. 1. Exact and asymptotic SER of fixed gain relaying for $N_1 = 1$, $N_2 = 2$, and $m_2 = 2$.

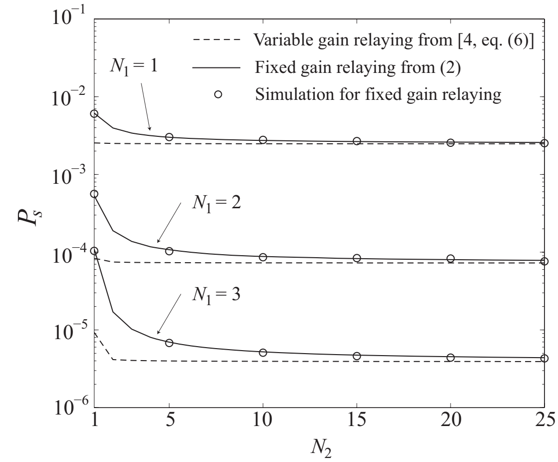

Fig. 2. Exact SER of fixed and variable gain relaying for $m_1 = 1$, $m_2 = 3$, and $E_S/N_0 = 20$ dB.

where $G_{a,V}$ is the array gain of variable gain relaying and $G_{a,F}$ is the array gain of fixed gain relaying.

To provide further insights into $G_{a,V}/G_{a,F}$, we examine the uplink and the downlink as two distinct scenarios in a typical cellular network. We first focus on the uplink, where the source and the destination act as a mobile terminal and a base station, respectively. In this scenario, we assume that $N_1 << N_2$ since the number of mobile terminal antennas is in general sufficiently less than the number of base station antennas. We also assume that $m_1 < m_2$ since the base station antennas and the relay station antennas are generally mounted on high towers with line-of-sight (LOS) conditions, whereas the mobile terminal is usually located in densely populated clusters with severe fading. Therefore, in the uplink with $N_1 m_1 << N_2 m_2$, $G_{a,V}/G_{a,F}$ reduces to

$$\frac{G_{a,V}}{G_{a,F}} \approx 1 + \frac{N_1}{N_2 \kappa}, \tag{15}$$

from which we note that $G_{a,V}/G_{a,F} \approx 1$ for large N_2. This reveals that in the uplink, fixed gain relaying yields the same SER as variable gain relaying in the large N_2 limit.

We then consider the downlink, where the source and the destination act as a base station and a mobile terminal, respectively. In this scenario, we assume that $N_1 m_1 >> N_2 m_2$ since the downlink and the uplink channels are reciprocal. As such, $G_{a,V}/G_{a,F}$ reduces to

$$\frac{G_{a,V}}{G_{a,F}} = \frac{N_1 m_1}{\sqrt[N_2 m_2]{\prod_{i=0}^{N_2 m_2} (N_1 m_1 - i)}}. \tag{16}$$

It is evident from (16) that $G_{a,V}/G_{a,F} \approx 1$ for $N_1 m_1 >> N_2 m_2$ (or equivalently $N_1 >> N_2$). Hence we remark that in the downlink, fixed and variable gain relaying exhibit the same performance in the large N_1 limit.

IV. Numerical Performance Comparison

In this section, numerical results are provided to examine the joint effects of beamforming and per-hop fading severities on fixed gain MIMO relaying. We assume equal average energies at the source and the relay with $E_S = E_R$. The exact SER results are substantiated via Monte Carlo simulations (marked with circles '$\circ$'). All our results are based on BPSK modulation. In each figure, our exact analytical curves match precisely with the simulations.

In Fig. 1, we plot the exact and asymptotic SER versus E_S/N_0 for various m_1. It is evident that our asymptotic results yield an excellent agreement with the "exact" curves at relatively high SNR. As predicted from (10), we see that when $m_1 \leq 4$, an increase in m_1 results in an increase in the diversity order, and hence a lower SER. However, an increase in m_1 from 4 to 5 has no effect on the diversity order, but still offers an SNR advantage due to the array gain.

In Fig. 2, we plot the exact SER of fixed and variable gain relaying versus N_2 for various N_1. We see that the performance gap between fixed and variable gain relaying substantially narrows as N_2 increases. For $N_2 > 20$, the SER of fixed gain relaying asymptotes to that of variable gain relaying, confirming our conclusions drawn in Section III-C.

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