A design procedure for finding optimal third order Delta-Sigma modulator loopfilters

GEORGI TSENOV[1], VALERI MLADENOV[1], JOSHUA D.REISS[2]
[1]Dept of Theoretical Electrical Engineering, [2]Dept of Electronic Engineering
[1]Technical University of Sofia, [2]Queen Mary, University of London
[1]BULGARIA, [2]UNITED KINGDOM
gogotzenov@tu-sofia.bg, valerim@tu-sofia.bg, josh.reiss@elec.qmul.ac.uk

Abstract: - In this paper we present the results obtained in our search for stable high performance single bit sigma-delta modulator loopfilter transfer functions. The results presented are made for third order modulators and give the performance impact on them when varying both poles and zeroes of the transfer function. This loopfilter function search is done with fast theoretical calculation of signal to noise ratio without need of simulations and combined with theory that gives approximated value for modulator's maximal stable DC input signal.

Key-Words: - sigma-delta modulators; digital signal processing; stability; analog-to-digital conversion

1 Introduction

Sigma-Delta modulators are the standard for analog to digital conversion nowadays (ADC). When using high oversampling ratios sigma-delta modulators (SDM) can achieve very high signal to noise ratio (SNR) [1]. They shape the noise and push it to frequencies higher than the operational band of interest. Thanks to its simplicity, single bit code shaping SDM are of greatest interest, because they're performance is influenced only by the loopfilter's transfer function and the modulator's oversampling ratio (OSR). Despite this in practice the modulator's maximal DC input signal range and its SNR are determined mostly by simulations, which also leave a zone of uncertainty. Furthermore a lot of engineers experiment with the loopfilter coefficients in order to achieve more SNR, but up to date there is still no such a thing as optimal loopfilter transfer function for specific modulator order that provides both high performance and stable modulator behavior. All of the realistic loopflter transfer functions have the poles grouped into a complex conjugate pairs and one real pole when having odd modulator order. In order to increase modulator performance some authors move one of the complex conjugate pair of poles [3] or the real pole [4] a little bit outside of the unit circle, while keeping the other poles inside resulting in increased SNR and reduced stability limit for maximal DC input signal amplitude beyond which the modulator becomes unstable.

This paper presents a design approach for a third order SDM taking into account the stability and SNR performance. The approach includes variation of the poles and zeroes positions on the unit circle of third order sigma-delta modulator loopfilter. For that type of analysis a parallel decomposition form of the loopfilter given in [2] is used. The results presented in [2] allows approximation of the maximal stable DC input signal value for single bit quantizer modulators with this particular filter form and they are used in the design procedure. For faster SNR calculation a derivation of it from the loopfilter noise transfer function is used, because in this case there is no need of modulator's output bitstream resulting in no need of SDM simulations [5].

The paper is organized as follows. In the next chapter a theoretical background necessary for understanding the approach is given. Then in the third chapter is presented the design procedure targeted to obtain SDM loopfilter transfer functions providing both decent performance and guaranteed stable modulator behavior. In the fourth chapter are presented the procedure results obtained from simulations and calculations. The conclusion remarks are given in fifth chapter.

2 Theoretical background

For better understanding the design approach here we will remind briefly the results in [2] that are used. The well known basic structure of an SDM is shown in Fig.1, and consists of a filter with transfer function G(z) followed by a one-bit quantizer in a feedback loop.

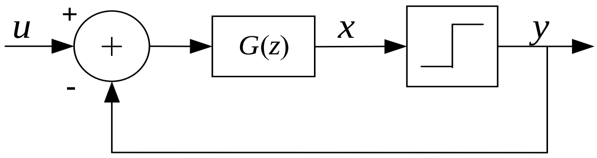

Figure1. Basic sigma delta modulator structure.

The system operates in discrete time and the input to the loop is a discrete-time sequence $u(n)\in[-1, 1]$, appearing in quantized form at the output. The discrete-time sequence $x(n)$ is output of the filter and quantizer input. Quantizer producing an output of +1 when its input is positive and –1 when its input is negative (single-bit), will not provide a good approximation to its input signal and for that reason an feedback loop is used, acting in such a way as to shift this quantization noise away from a certain frequency band. If an input signal from within this frequency band is applied to the loop, most of the noise imposed by the quantization process will lie outside the frequency band of interest and can subsequently be filtered out, leaving a good approximation to the input signal. This process is called noise shaping.

In [2] authors consider a N^{th} order modulator with a loop filter transfer function of the form

$$G(z) = \frac{a_1 z^{-1} + \ldots + a_N z^{-N}}{1 + d_1 z^{-1} + d_2 z^{-2} + \ldots + d_N z^{-N}} \quad (1)$$

In the general case the loop filter transfer function have complex conjugated roots. Without loss of generality we will consider only one pair of complex conjugated roots. In this case (1) becomes

$$G(z) = \frac{b_1 z^{-1}}{1-\lambda_1 z^{-1}} + \ldots + G_2(z) = \frac{b_1 z^{-1}}{1-\lambda_1 z^{-1}} + \ldots \frac{B_{N-1} z^{-1} + B_N z^{-2}}{1 - d_1 z^{-1} - d_2 z^{-2}} \quad (2)$$

where the coefficients b_i, $i=1,2,\ldots,N$ of the fractional components can be found easily using the well known formula $b_i = \left.\frac{(1-\lambda_i z^{-1})}{z^{-1}} G(z)\right|_{z=\lambda_i}$.

The denominator of the last part of (2) has a complex conjugated pair of roots and therefore (2) becomes

$$G(z) = \frac{b_1 z^{-1}}{1-\lambda_1 z^{-1}} + \ldots + \frac{b_{N-1} z^{-1}}{1-\lambda_{N-1} z^{-1}} + \frac{b_N z^{-1}}{1-\lambda_N z^{-1}} \quad (3)$$

where

$$\lambda_{N-1} = \alpha + j\beta,\ \lambda_N = \alpha - j\beta$$
$$b_{N-1} = \delta - j\gamma,\ b_N = \delta + j\gamma \quad (4)$$

i.e. λ_{N-1}, λ_N and b_{N-1}, b_N are complex conjugated numbers.

Because of this in [2] the parallel presentation given in Fig.2 of third order modulator is used. The values of the last two blocks are complex, but the output signal of these two blocks is real. They correspond to a second order SDM with complex conjugated poles of the loop filter transfer function $G(z)$. Both signals x_2 and x_3 are complex conjugated (and x_2 is real), namely

$$x_2(k+1) = m(k+1) + jn(k+1) \quad (5)$$
$$x_3(k+1) = m(k+1) - jn(k+1)$$

Because of this the input of the quantizer is real i.e.

$$(\delta - j\gamma)x_2(k) + (\delta + j\gamma)x_3(k) = 2\delta m(k) + 2\gamma n(k) \quad (6)$$

The modulator could be considered as three first order modulators interacting only through the quantizer function. The connected signals with two modulators are complex, but the input and output signals (u and y) are the "true" signals of the modulator. As it is stressed in [2] both modulators work cooperative, because their signals are conjugated. These modulators do not exist in the real SDM and they are introduced to help the analysis of the behavior of the whole system.

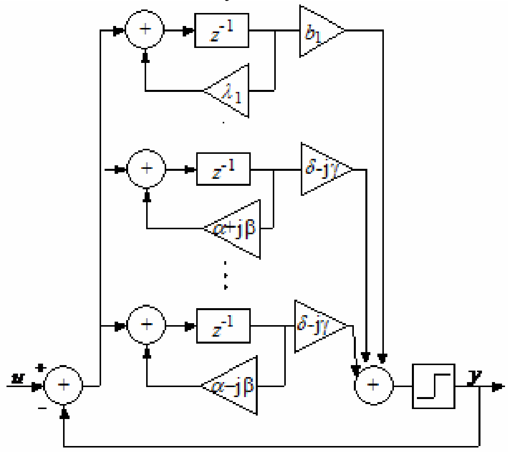

Figure2. Block diagram of third order SDM with parallel loopfilter form

The benefit of this modulator representation is because we can determine whenever the modulator is stable or not by the following criterion [2]

$$\frac{(2-\lambda_1)}{\lambda_1}\frac{b_1}{(\lambda_1 - 1)} > -\sum_{i=2}^{N-2}\frac{|b_i|}{\lambda_i - 1} + \frac{2\,|\,\delta(1-\alpha)+\gamma\beta\,|}{(1-\alpha)^2+\beta^2} \quad (7)$$

and additionally we can also determine the maximal range of input signal ensuring the stability expressed by Δu (DeltaU)

$$\Delta u < \frac{\sum_{i=2}^{N-2}\frac{|b_i|}{\lambda_i - 1} - \frac{2|\delta(1-\alpha)+\gamma\beta|}{(1-\alpha)^2+\beta^2} + \frac{b_1(2-\lambda_1)}{\lambda_1(\lambda_1 - 1)}}{\frac{b_1}{\lambda_1 - 1} - \sum_{i=2}^{N-2}\frac{|b_i|}{\lambda_i - 1} + \frac{2|\delta(1-\alpha)+\gamma\beta|}{(1-\alpha)^2+\beta^2}} \quad (8)$$

3 Design Approach

For third order modulator with the realistic form of the loopfilter (third order transfer function with one real and two complex conjugate poles) a pole variation is performed (Fig.3). This is done by moving the complex conjugate pair inside the unit circle and the real pole on the unit circle and away from it in logarithmic increments. In order to have result from stability condition (7) and Δu (8), these equations allows only one of the poles to be outside of the unit circle and this is the reason why the complex conjugate pair is not moved outside the unit circle. Some authors prefer to move the complex conjugate pair outside of the unit circle, because they end up with higher SNR, but with decreased range of maximal acceptable input signal [4]. In these cases at some point when increasing the input signal amplitude the modulator behavior becomes unstable and maximum input signal clipping limits determined by simulations are used in order to keep the modulator stable. On the other hand moving only the real pole produces higher SNR (but not as high as when moving complex pairs), while keeping the maximal range of input signal almost equal to 1 [3]. Moving the real pole too far away from the unit circle produces decrease of SNR and loss of stability and for that reason we moved only the real pole outside the unit circle. Also when moving the real pole inside of the unit circle, when having all other poles are also inside leads to SNR decrease.

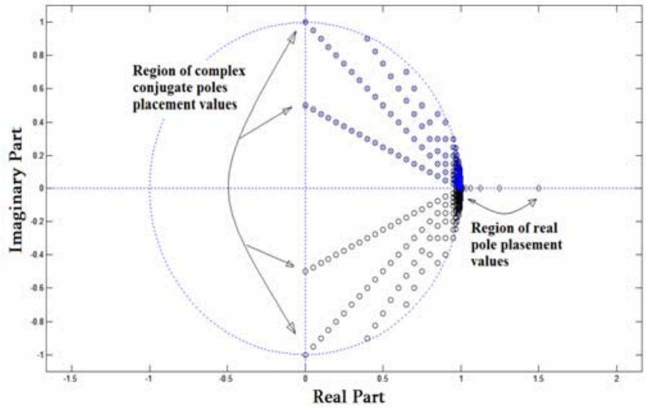

Figure 3. Position placements of the loopfilter's poles

The modulus of the complex pole positions is shown in Figure 4. Values equal to almost 1 means that the complex pair is very close and almost lying on the unit circle.

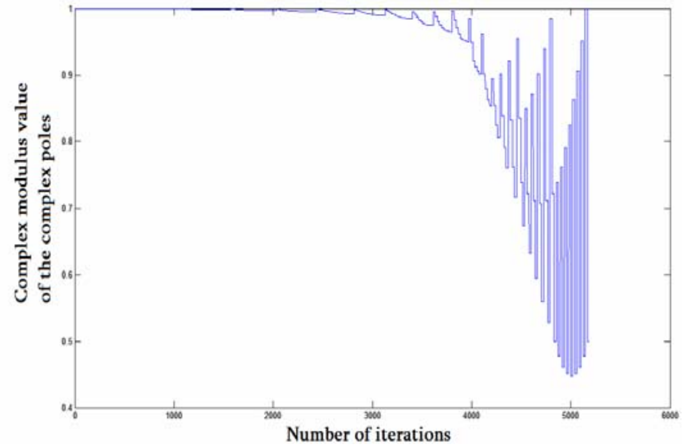

Figure 4. Modulus of the complex poles values for all the complex pole placement positions on the unit circle

The filter design procedure also includes usage of two real zeroes and variation of their values on the real axis. For one loopfilter case, where the zeros are chosen to be two and real values for some fixed zero positions and the pole positions varied, the results for SNR value are given in Fig.5 for 64 times oversampling ratio (OSR), input sine wave with scaled amplitude value 0.5 and frequency 2/3 of our band of interest, when SNR is calculated from simulations. Also for comparison in Fig.6 is shown SNR values that are calculated and theoretically derived from loopfilter transfer functions [5] that are formed at every iteration step.

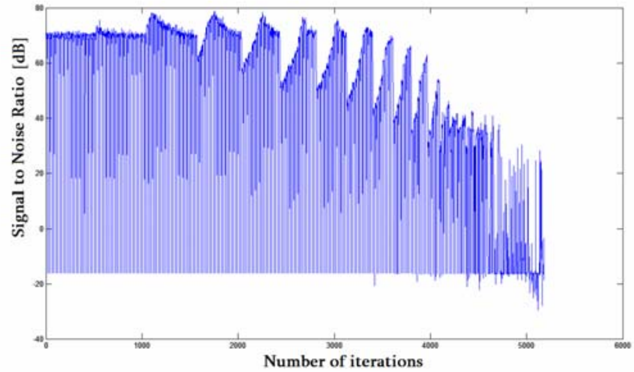

Figure 5. SNR results calculated from simulations for fixed zeros combination and variation of the real and complex conjugate poles

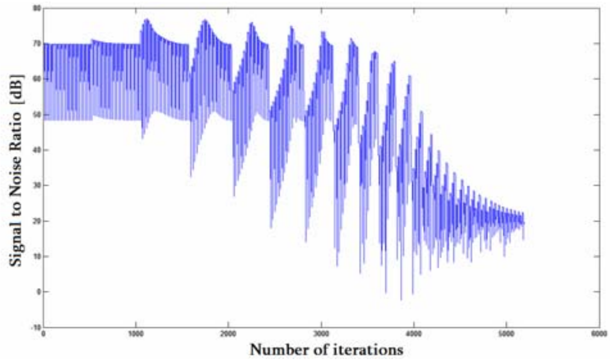

Figure 6. SNR results calculated on derivations from the loopfiler transfer function for fixed zeros combination and variation of the real and complex conjugate pair poles

On Fig.7 for this same particular case the theoretically derived for every iteration Δu value is also plotted.

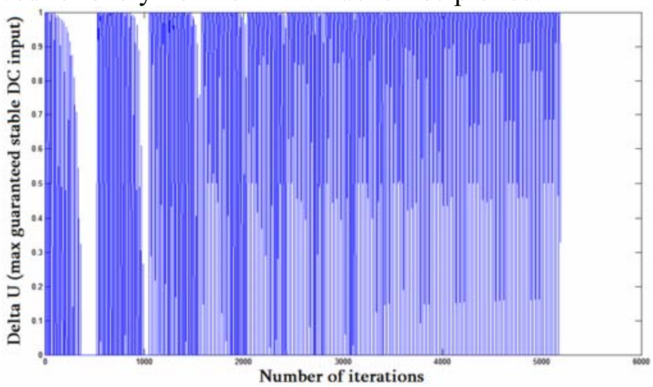

Figure 7. DeltaU values for one fixed zeros combination and variation of the real and complex conjugate poles

The practical relation of Δu and modulator stability depending on its signal value is given in Fig.8 in one example with usage of poor loopfilter transfer function that produces loss of stability for input signals with higher amplitude. Here when rising the test sine wave amplitude we can observe that at some point we start to have an SNR decrease and eventually loss of stability. This means that the modulator can be stable for input signals with value higher than that of Δu, but then we can't guarantee its stability (Δu=0.68 in this example).

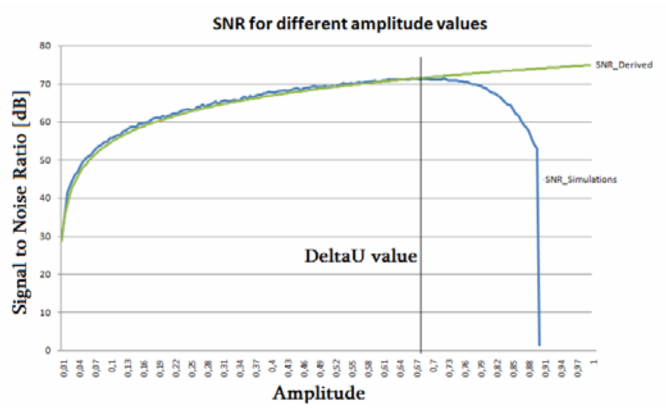

Figure 8. Relation between DeltaU and the value of the input signal

4 Simulation Results

A procedure rotating all the poles and having two real zeros for additional movement inside the unit circle was performed. Because of the reduced time for SNR calculations when using the approach for deriving it from the loopfilter transfer function was used theoretical SNR calculation. The theoretical SNR derivation showed very high SNR values (as high as more than 100dB for third order modulator) when the zeros had small values and because the poles were inside the unit circle Δu criterion had also a good value. In practice however, when simulation was performed on SDM model when using these candidate functions we end up with modulator behavior that does not correspond to SDM, i.e. the modulator was unstable. When moving the zeroes to positions greater than 0.65 then the modulator becomes stable. Additionally we observed that moving the zeros to values beyond 0.8 leads to decrease of SNR and loss of stability when moving the zeroes to much higher values. Also when having the zeroes with this bigger values the formula for Δu result appears to be correct and the calculated from loopfilter transfer function derivations SNR is almost equal to the one calculated from simulations. For this reason the search procedure was modified to start the zero movement from points with values 0.65 for the zeroes and stop at 0.8 (SNR results on Fig.9).

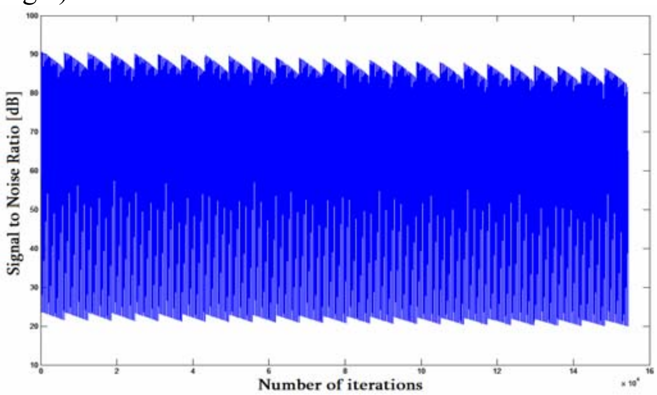

Figure 9. Combined zero and pole variation and corresponding SNR

All the results presented are given for 64 times OSR and amplitude value 0.5 of the test input signal sine wave with frequency equal to 2/3 of the band of interest. This modified procedure give back as an result a lot of transfer functions that give an modulator SNR little over or little less around 90 dB. SNR of 90dB for third order loopfilter transfer function at 64 times OSR is more than the 83dB provided by used for comparison DStoolbox optimized transfer function, especially when the acquired transfer functions have Δu value almost equal to one. One curious observation we had was that when moving the complex pole pair deep inside the unit circle the modulator vastly decreases its performance and in some cases even loses his stability as can be seen on Fig.5 and Fig6, where the last iterations include the poles nearing the imaginary axis. One example of modulator power spectrum shape obtained after simulations, when using test sine wave for modulator with one of the optimal loopfilter transfer functions is shown on Fig.10. One such a transfer function is with two zeros at points 0.76 and 0.7 lying on the real axis, and with poles equal to 1.000001907348633, 0.9985+0.039i and 0.9985-0.039i. For this example we get value for max stable DC input at Δu=0.999957508095839, or we can say that the modulator is stable up to input signals with value almost 1. For this single example the poles and zeros form loopfilter transfer function G(z) having the following polynomial form:

$$G(z)=\frac{z^2-1.46z+0.532}{z^3-2.997001907348633z^2+2.995527058975220z-0.998525154531956} \quad (9)$$

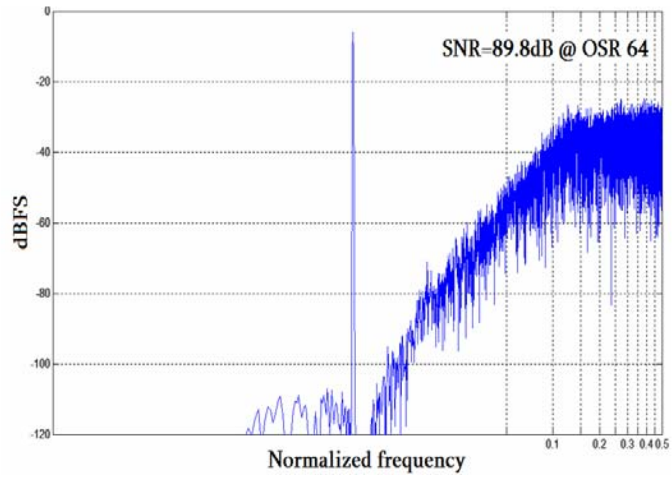

Figure 10. SDM spectrum for third order loopfilter transfer function providing 90dB SNR

5 Conclusion

In the paper a design approach for stable high performance sigma-delta modulator is presented. Based on this approach a third order stable SDM with reasonable performance in sense of SNR and stable DC input signal range is obtained. The approach easily can be generalized for high order modulators. The loss of stability when modulator loopfilter poles and zeros are deep inside the unit circle and extension of the search procedure for higher optimal modulator loopfilter order should be a topic for further research.

Acknowledgment

This work was supported by a Royal Society International Joint Project, "Extended Theory and Design of High-order Sigma-Delta Modulators."

References:

[1] R. Schreier and G. C. Temes, Understanding Delta-Sigma Data Converters. New Jersey: John Wilet & Sons, 2005.

[2] V. Mladenov, H. Hegt, and A. v. Roermund, "On the Stability Analysis of Sigma-Delta Modulators," 16th European Conference on Circuit Theory and Design ECCTD 2003, Cracow, Poland, pp. I-97-I-100, 2003.

[3] G. Tsenov, V. Mladenov, and J.D.Reiss, "A Comparison of Theoretical, Simulated, and Experimental Results Concerning the Stability of Sigma Delta Modulators", 124th AES Convention, May 2008.

[4] D. Reefman and E. Janssen, "Signal processing for Direct Stream Digital: A tutorial for digital Sigma Delta modulation and 1-bit digital audio processing," Philips Research, Eindhoven, White Paper 18 December 2002.

[5] J.D.Reiss, "Understanding sigma delta modulation: the solved and unsolved issues," Journal of the Audio Engineering Society, vol. 56., 1/2, p.49-64, Jan./Feb. 2008.