

[Home Page](#)

[Title Page](#)



Page 1 of 24

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

# Fibrational Structures in Causal and Nonmonotonic Reasoning

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## Problem I: Vocabulary Change

Consider the orthodox treatment of the frame problem:

1. a language with
  - *situations*
  - *fluents* (propositional atoms parametrised by situations)
  - *actions* (think of these as labelled transitions between situations)
2. merely axiomatising the effects of axioms underspecifies models
  - it doesn't tell us about the fate of fluents not affected by actions

Home Page

Title Page



Page 2 of 24

Go Back

Full Screen

Close

Quit

Home Page

Title Page



Page 3 of 24

Go Back

Full Screen

Close

Quit

3. one resolution due to David Lewis:

- give yourself a relation between situations which allows you to say how *close* they are to each other
- then the result of an action is that situation which
  - is compatible with the results of actions
  - and otherwise is closest to the starting situation

4. plus an extra ingredient (McCarthy):

- closeness is measured by *how many fluents change*

## The Problem

- this is badly vocabulary dependent:
  - we can have two different, but equivalent, languages
  - equivalent axiomatisations in each language
  - which give different results
  - in fact, if there are only a finite number of models, we can get *any closeness order we like* by suitable choice of vocabulary
- the problem is the concept of a *fluent*:
- these are propositional *atoms*, hence vocabulary dependent

Home Page

Title Page



Page 4 of 24

Go Back

Full Screen

Close

Quit

Home Page

Title Page



Page 5 of 24

Go Back

Full Screen

Close

Quit

## Responses

- Makinson <sup>1</sup> argues that nonmonotonic consequence relations are not closed under uniform substitution
- and that they do not need to be
- and an example would be this sort of language dependence
- but this is surely fatal for *applications* of the theory:
  - we have to worry about more than mere correctness
  - but also whether we have the right magic vocabulary

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<sup>1</sup>([Makinson, 2003](#), pp. 10f.)

## However:

- we have the beginnings of a fibrational structure
  - contexts (i.e. objects of the base category) are languages
  - fibres are the nonmonotonic theories
  - morphisms are equivalences
  - pullback is given by translation
- the history of category theory tells us that it is a good idea to look at all morphisms, not just isomorphisms:
- so we wonder what the *other* morphisms of the base category might be

Home Page

Title Page



Page 6 of 24

Go Back

Full Screen

Close

Quit

## Problem 2: Progression and Regression

### Formulation 1

(This work is mostly due to Reiter<sup>2</sup> and his school)

### The Base

- Suppose we start with a base category of situations
- or rather situation descriptions: i.e. situations + languages
- it is traditional in AI to think of the situations as tree-like
  - this is what they call the “unique names assumption”
  - which means that you consider the initial model of everything in sight
- but we can think of it simply as a labelled transition system

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<sup>2</sup>([Lin & Reiter, 1994](#))

Home Page

Title Page



Page 7 of 24

Go Back

Full Screen

Close

Quit

Home Page

Title Page



Page 8 of 24

Go Back

Full Screen

Close

Quit

## The Fibres

- the fibres are where the logic lives:
- the fibre over a situation should contain enough resources to describe that situation
  - a language
  - a consequence relation on that language
  - a final object, so that the set of theorems should be the set of propositions true in that situation
- note that a fibre contains information *only about that situation*

## The Pullbacks

- suppose we have  $s \xrightarrow{\alpha} t$
- then, given a proposition  $P$  at  $t$ , we can hope to find a proposition  $\alpha^*(P)$  at  $s$  such that
$$\vdash_s \alpha^*(P) \text{ iff } \vdash_t P$$
- in *dynamic logic* (and if there is only one  $\alpha$  transition from  $s$ )  $\alpha^*P$  corresponds to  $[\alpha]P$
- and there is a dynamic logic version of these things in Zhang and Foo<sup>3</sup>
- but we will describe work by Waldinger<sup>4</sup>, Pednault<sup>5</sup> and Reiter<sup>6</sup>

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<sup>3</sup>(Zhang & Foo, 2001),(Zhang & Foo, 2002)

<sup>4</sup>(Waldinger, 1977)

<sup>5</sup>(Pednault, 1986), (Pednault, 1988), (Pednault, 1989)

<sup>6</sup>(Reiter, 1991),(Reiter, 2001)

## Specifying Actions

- suppose that  $\alpha$  has the following effect:

before	after
$\phi_1$	$\psi_1$
$\vdots$	$\vdots$
$\phi_k$	$\psi_k$

where the  $\phi$ s and  $\psi$ s are conjunctions of fluents

- note that the  $\phi$ s can be incompatible, which allows for context dependent effects
- how does this specify the truth values of propositions at  $t$ ?
- by *explanatory closure*:

$P$  is true at  $t$  iff

1. it is entailed by a conjunction of  $\psi_i$ s, and the corresponding  $\phi_i$ s are true in  $s$ , or
2.  $P$  is true in  $s$ , and compatible with everything of type 1, or
3. entailed by a conjunction of propositions of types 1 and 2.

Home Page

Title Page



Page 10 of 24

Go Back

Full Screen

Close

Quit

## Indeterminacy

- note that this recipe may have more than one solution:

**Example 1** *Let  $s$  make  $P, Q,$  and  $R$  true, and suppose that the action  $\alpha$  has exactly one rule:*

<i>before</i>	<i>after</i>
$P$	$\neg Q \vee \neg R$

*then there are two explanatorily closed states  $t$ : one makes  $Q$  false,  $P$  and  $R$  true, and the other makes  $R$  false,  $P$  and  $Q$  true.*

- and there may be no solutions, if the propositions in category 1 are inconsistent
- so, for generality and simplicity, we will consider the case of a non-deterministic action  $\alpha$ : that is, there may be a number (including zero) of transitions from  $s$  to different states  $t$ , all labelled by  $\alpha$
- the category 1 propositions are the same for all of them
- and we will specialise to the case of deterministic actions at the end

Home Page

Title Page



Page 11 of 24

Go Back

Full Screen

Close

Quit

## The Direct Effects

- these are the propositions in category 1 above

- let

$$\Box_0 P = \bigvee_{\psi_{i_1}, \dots, \psi_{i_k} \vdash P} \phi_{i_1} \wedge \dots \wedge \phi_{i_k}$$

- then

$\Box_0 P$  is true at  $s \Leftrightarrow P$  is in Category 1 for any  $\alpha$  transition from  $s$

- ( $\Leftarrow$  is clear:  $\Rightarrow$  takes more argument)

Home Page

Title Page



Page 13 of 24

Go Back

Full Screen

Close

Quit

## Inertia

- these are the propositions in category 2 above
- let  $\diamond_0 P = \neg \square_0 \neg P$
- we can show that

$\diamond_0 P$  is true at  $s \Leftrightarrow P$  is in category 2 for *some*  $\alpha$  transition from  $s$

## Finally

- let

$$\diamond P = \bigvee_{X, Y \vdash P} \Box_0 X \wedge Y \wedge \Diamond_0 Y$$

- this is precisely the dynamic logic  $\langle \alpha \rangle$
- the first conjunct corresponds to category 1, the second and third to category 2
- and so  $[\alpha] = \neg \langle \alpha \rangle \neg$
- in the deterministic case,  $[\alpha]$  and  $\langle \alpha \rangle$  coincide
- in this case, we let  $\alpha^*$  be their common value

[Home Page](#)

[Title Page](#)



Page 15 of 24

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

## Formulation 2

- this is OK, but
  1. it assumes that we somehow have a grasp of what all the objects of the transition system are
  2. and it leaves undescribed the entailment relation in the fibres
  3. so here's another description

## The Base

- as before

## The Fibres

- the language consists of fluents instantiated at that state
- the entailment relation is classical entailment
- we also have an axiom,  $\Xi_s$ , consisting of the set of fluents and their negations true in that state
- morphisms between  $P$  and  $Q$  in the fibres are entailments

$$\Xi_s, P \vdash Q$$

- this is *equivalent* to the previous description
- but it is more amenable to calculation: we can work out the  $\Xi_s$

- If we have  $s \xrightarrow{\alpha} t$

$$\Xi_s \vdash \alpha^* P \quad \text{iff} \quad \Xi_t \vdash P$$

so

$$\Xi_t \equiv \exists_{\alpha^*} \Xi_s$$

where  $\exists_{\alpha^*}$  is the left adjoint to  $\alpha^*$ .<sup>7</sup>

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<sup>7</sup>(Lawvere, 1969), (Lawvere, 1970)

Home Page

Title Page



Page 17 of 24

Go Back

Full Screen

Close

Quit

- this is second order in general, but in practical cases one can find closed form expressions for the result<sup>8</sup>
- this approach can be highly efficient: planners based on goal regression can do very well, because they can prune the search by using domain constraints<sup>9</sup>

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<sup>8</sup>(Lin & Reiter, 1994),(Reiter, 2001, Chapter 9)

<sup>9</sup>(Gabaldon, 2003)

## Relation with Reiter's Work

- Reiter's great insight is to realise that the contravariant operators ( $[\alpha], \langle \alpha \rangle$ ) are *much easier* to specify than the covariant ones ( $\exists_\alpha$ ), and that it's a good idea to start with the contravariant ones
- The work of Reiter and his colleagues is limited to formulations without domain constraints: ours is not
- they also work solely in the deterministic case, and also assume that one can separate a proposition which gives a condition for the success of an action (i.e.  $\langle \alpha \rangle \top$ )
- under these circumstances,  $[\alpha] = \langle \alpha \rangle$ , and so both operators commute with conjunctions and disjunctions
- they can then be specified by giving their effect on atoms, which is what they do
- in the nondeterministic case, however,  $[\alpha]$  commutes with conjunctions (and thus has a left adjoint), whereas  $\langle \alpha \rangle$  commutes with disjunctions: neither can be given by their effect on atoms.

Home Page

Title Page

◀ ▶

◀ ▶

Page 18 of 24

Go Back

Full Screen

Close

Quit

## Problem 3: Interpolation and Decomposing Situations

- Zhang and Foo<sup>10</sup> prove an interpolation theorem for their treatment of the frame problem
- in our terms, this goes as follows: suppose that we have  $s \xrightarrow{\alpha} t$ , and suppose
  - we can decompose the vocabulary into a small part  $\mathcal{L}'$ , containing the action  $\alpha$  and the fluents relevant to that action
  - and a large part  $\mathcal{L}''$ , with fluents only

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<sup>10</sup>(Zhang & Foo, 2002)

[Home Page](#)[Title Page](#)[◀](#) [▶](#)[◀](#) [▶](#)[Page 20 of 24](#)[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)

- this decomposes  $\Xi_s$  and  $\Xi_t$  similarly:

$$\begin{aligned}\Xi_s &= \Xi'_s \wedge \Xi''_s \\ \Xi_t &= \Xi'_t \wedge \Xi''_t\end{aligned}$$

and we have

$$\begin{aligned}\Xi'_s \vdash \alpha^* P' &\text{ iff } \Xi'_t \vdash P \\ \Xi''_s \vdash P'' &\text{ iff } \Xi''_t \vdash P\end{aligned}$$

where  $P' \in \mathcal{L}'$  and  $P'' \in \mathcal{L}''$ .

- so  $\Xi''_s \equiv \Xi''_t$ , and the only seriously nonmonotonic part of this has to do with  $\mathcal{L}'$ , which is small.
- this makes the problem hugely more tractable
- interpolation-based problem decomposition seems in general to be a good strategy<sup>11</sup>

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<sup>11</sup>(Amir & Engelhardt, 2003), (Amir & MacIraith, 2000)

## What Structure in the Base?

- this would correspond to a *monoidal* structure in the base category
- but we probably don't want products:
- these systems are very sensitive to adding or removing propositions from the vocabulary
- so weakening is bad
- so: some sort of relevance logic in the base?

Home Page

Title Page



Page 21 of 24

Go Back

Full Screen

Close

Quit

## References

- Amir, Eyal, & Engelhardt, Barbara. 2003. Factored Planning. *Pages 929–935 of: Fifteenth International Joint Conference on Artificial Intelligence (IJCAI'03)*. IJCAI. pages 20
- Amir, Eyal, & Maclraith, Sheila. 2000. Partition-Based Logical Reasoning. *Pages 389–400 of: Proceedings of KR'00*. Morgan Kaufmann. pages 20
- Gabalton, Alfredo. 2003. Compiling Control Knowledge into Preconditions for Planning in the Situation Calculus. *Pages 1061–1066 of: Fifteenth International Joint Conference on Artificial Intelligence (IJCAI'03)*. IJCAI. pages 17
- Lawvere, F.W. 1969. Adjointness in Foundations. *Dialectica*, 23, 281–296. pages 16
- Lawvere, F.W. 1970. Equality in Hyperdoctrines and the Comprehension Scheme as an Adjoint Functor. *Pages 1–14 of: Heller, A. (ed), Applications of Categorical Algebra*. Providence RI: AMS. pages 16
- Lin, Fangzhen, & Reiter, Raymond. 1994. How to Progress a Database (and Why): I. Logical Foundations. *Pages 425–436 of: Doyle, Jon, Sandewall, Erik, & Torasso, Pietro (eds), Proceedings of KR'94: The*

Home Page

Title Page



Page 22 of 24

Go Back

Full Screen

Close

Quit

*Fourth International Conference on the Principles of Knowledge Representation and Reasoning.* pages 7, 17

Makinson, David. 2003. Ways of Doing Logic: What was Different about AGM 1985? *Journal of Logic and Computation*, 13(1), 3–13. pages 5

Pednault, Edwin. 1989. ADL: Exploring the Middle Ground Between STRIPS and the Situation Calculus. *Pages 324–332 of: KR'89.* pages 9

Pednault, Edwin P. 1986. *Toward a Mathematical Theory of Plan Synthesis.* Ph.D. thesis, Department of Electrical Engineering, Stanford University, Stanford CA. pages 9

Pednault, Edwin P. 1988. Synthesising Plans that Contain Actions with Context-Dependent Effects. *Computational Intelligence*, 357–372. pages 9

Reiter, Raymond. 1991. The Frame Problem in the Situation Calculus: A Simple Solution (Sometimes) and a Completeness Result for Goal Regression. *Pages 418–420 of: Lifschitz, V.I. (ed), Artificial Intelligence and Mathematical Theory of Computation: Papers in Honour of John McCarthy.* San Diego CA: Academic Press. pages 9

Reiter, Raymond. 2001. *Knowledge in Action: Logical Foundations for Specifying and Implementing Dynamical Systems.* Cambridge MA: MIT Press. pages 9, 17

Home Page

Title Page



Page 23 of 24

Go Back

Full Screen

Close

Quit

[Home Page](#)[Title Page](#)

Page 24 of 24

[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)

Waldinger, R. 1977. Achieving Several Goals Simultaneously. *Pages 94–136 of: Elcock, E., & Michie, D. (eds), Machine Intelligence.* Edinburgh: Ellis Horwood. pages 9

Zhang, Dongmo, & Foo, Norman. 2001. EPDL: A Logic for Causal Reasoning. *Pages 131–137 of: Nebel, Bernhard (ed), IJCAI-01: Proceedings of the Seventeenth International Joint Conference on Artificial Intelligence.* San Francisco, CA: Morgan Kaufman, for International Joint Conferences on Artificial Intelligence, Inc. and The American Association for Artificial Intelligence. pages 9

Zhang, Dongmo, & Foo, Norman. 2002. Interpolation Properties of Action Logic: Lazy-Formalization in the Frame Problem. *In: Proceedings of JELIA2002.* Springer. pages 9, 19