

On the Expressive Power of Polyadic Synchronisation in π -calculus

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Intro

π -calculus vs. locality based calculi like:

- D- π , Isd- π , etc.
- Ambient Calculus (?)
- etc.

Intro

- We can obtain a new locality based calculus by simply extending π -calculus syntax into ${}^e\pi$.
- We need to extend the channel name to a *vector of names* like:

$$\overline{a_1 \cdot \dots \cdot a_n} \langle b \rangle . P \mid b_1 \cdot \dots \cdot b_n(x) . Q$$

* Only if $a_i = b_i \ \forall i$

Intro

What do we get?

We get:

- Simple way of expressing localities.
- New expressive power.
- π -calculus theory.

Plan

- Example: D- π
- The Calculus
- The Matching Problem
- Expressiveness
- $^e\pi$ vs. π
- Encoding Local Area π -calculus
- Conclusion and Future Work

Example: D- π

Distributed π -calculus (Hennessy - Riely)

$$(rcomm) \quad l[a\langle v \rangle!P] \mid l[a(x)?Q] \mapsto l[P] \mid l[Q\{v/x\}]$$

$$\llbracket l[a(m)!P \mid a(x)?go\ x.b(v)!Q] \mid m[b(y)?R] \rrbracket$$

=

$$\overline{l} \cdot a\langle m \rangle.P \mid l \cdot a(x).\overline{x} \cdot b\langle v \rangle.Q \mid m \cdot b(y).R$$

$e\pi$: Syntax and Semantics

- Syntax

$$P, Q, R ::= 0 \mid \Sigma_i \alpha_i.P_i \mid P \mid Q \mid (\nu n)P \mid !P$$

$$\alpha ::= a_1 \cdot \dots \cdot a_k(x) \mid \overline{a_1 \cdot \dots \cdot a_k} \langle m \rangle.$$

- Semantics

$$\frac{}{\Sigma_i \alpha_i.P_i \xrightarrow{\alpha_i} P_i} \quad (\text{PREFIX}) \quad \frac{P \xrightarrow{\bar{u} \langle b \rangle} P', Q \xrightarrow{u(x)} Q'}{P \mid Q \xrightarrow{\tau} P' \mid Q' \{b/x\}} \quad (\text{COM})$$

$e\pi$: Syntax and Semantics

$$\frac{P \mid !P \xrightarrow{\alpha} P'}{!P \xrightarrow{\alpha} P'} \quad (\text{BANG}) \quad \frac{P \xrightarrow{\alpha} P'}{(\nu x)P \xrightarrow{\alpha} (\nu x)P'} \quad x \notin \alpha \quad (\text{RES})$$

$$\frac{P \xrightarrow{\bar{u}\langle x \rangle} P'}{(\nu x)P \xrightarrow{\bar{u}\langle \nu x \rangle} P'} \quad x \notin u \quad (\text{OPEN})$$

$$\frac{P \xrightarrow{\bar{u}\langle \nu x \rangle} P', Q \xrightarrow{u(x)} Q'}{P \mid Q \xrightarrow{\tau} (\nu x)(P' \mid Q')} \quad (\text{CLOSE})$$

Example: Matching in π_2

$$\llbracket [a = b]P \rrbracket = (\nu n)(\overline{a \cdot n} \langle n \rangle \mid b \cdot n(x).P)$$

where x, n are not free in P .

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Expressiveness: The Matching Problem

Process Template.

P is a *process template of degree n* if and only if its free names (parameters) are $\{y, x_1, \dots, x_n\}$.

We write it as $P_{(x_1, \dots, x_n)}^{(y)}$ (or simply as P according to the context).

Example.

$P = \overline{x_1 \cdot x_2} \langle x_2 \rangle \mid x_1 \cdot x_2(w). \langle y \rangle$ can be seen as a process template $P_{(x_1, x_2)}^{(y)}$ of degree 2.

Expressiveness: The Matching Problem

Matching System.

Given K and L of degree n , $MS_n(K, L)$ is a Matching System of arity n iff for all P of the form (given I and J)

$$\prod_{i \in I} K_{k_1^i, \dots, k_n^i}^i \mid \prod_{j \in J} !L_{l_1^j, \dots, l_n^j}^j$$

we have that (after k reductions such that P cannot reduce anymore):

Expressiveness: The Matching Problem

- each instance i of K is associated with a matching instance j of L (if any exists).

$$P \Longrightarrow \xrightarrow{\langle i,j \rangle}$$

- in all the matched pairs, the corresponding instances of K and L must have the same identifiers.
- There is no infinite sequence of reductions starting from process P .

Expressiveness: The Matching Problem

Example.

The processes $K_{()}^{(y)} = \langle y \rangle$ and $L_{()}^{(y)} = (w). \langle w, y \rangle$ constitute a matching system $MS_0(K, L)$.

K and L have no identifiers \Rightarrow every exchange of indices is legal.

For example:

$$\langle i^1 \rangle | \langle i^2 \rangle | \langle i^3 \rangle | !(w). \langle w, j \rangle \Longrightarrow \langle i^1, j \rangle | \langle i^2, j \rangle | \langle i^3, j \rangle | !(w). \langle w, j \rangle \not\Longrightarrow$$

and conditions of the definition are satisfied.

Expressiveness: The Matching Problem

Matching Problem.

The Matching Problem MP_m of arity m consists in finding two processes K, L such that $MS_m(K, L)$.

Expressiveness: Main Result

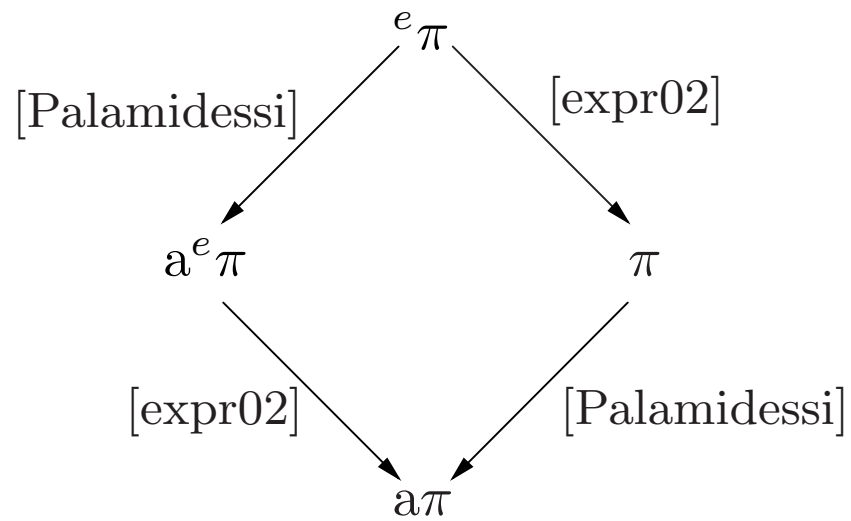
Theorem. For all nonnegative integer numbers n and m the problem MP_m has a solution in π_n if and only if $n \geq m$.

Expressiveness: Corollary

The next three results are direct consequences of the Theorem

1. The $\pi_{\mathcal{N}}$ family is strictly ordered by inclusion:
 $\pi_0 \subset \pi_1 \subset \dots \subset {}^e\pi$.
2. It is impossible to have a *reasonable* encoding of channels in π_0 .
3. The ${}^e\pi$ is strictly more expressive than the π -calculus, in particular there exists no *reasonable* encoding of ${}^e\pi$ in π -calculus.

Expressiveness: Main Framework



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^e π VS. π

Encoding into π :

$$\llbracket \overline{b \cdot a} \langle c \rangle \rrbracket = \bar{b} \langle a, c \rangle \quad (\text{OUT})$$

$$\llbracket b \cdot a(y).P \rrbracket = \mu X. b(x, y). [x = a] \llbracket P \rrbracket, (\bar{b} \langle x, y \rangle \mid X) \quad (\text{INP})$$

Theorem. $P \dot{\sim} Q$ if and only if $\llbracket P \rrbracket \dot{\sim}_\pi \llbracket Q \rrbracket$.

Encoding Local Area π -calculus

$$\llbracket \Gamma \vdash_l m[P] \rrbracket_{\Delta} = (\nu e) \llbracket \Gamma \vdash_m P \rrbracket_{\Delta, m \mapsto e} \quad e \notin fn(P) \cup cod(\Delta) \quad (\text{AREA})$$

$$\llbracket \Gamma \vdash_l \bar{a}\langle b \rangle \rrbracket_{\Delta} = \overline{e \cdot a}\langle b \rangle \quad (\text{OUT})$$

$$\llbracket \Gamma \vdash_l a(b).P \rrbracket_{\Delta} = e \cdot a(b). \llbracket \Gamma, b : \sigma \vdash_l P \rrbracket_{\Delta} \quad (\text{IN})$$

Conclusions

- We have extended the communication mechanism of π -calculus.
- We have shown by the existence of a separation problem that this extension is not trivial and implies:
 - an interesting class of problems arising in distributed systems to be solved in the π -calculus framework.

Conclusions

- calculi with an explicit notion of location are to some extent more expressive than the π -calculus.
- We have given an example of such a problem, by encoding *local areas* in ${}^e\pi$.

Future Work

- To model encryption.
- To investigate on other distributed calculi ($D\pi$, Ambients?).
- Dynamic binding (Vivas).
- Join Calculus.

Thank You!