Blind Signal Separation of Convolutive Mixtures

P D Baxter and J G McWhirter
QinetiQ, Malvern
Blind Signal Separation (instantaneous)

- Signal model
  \[ x(t) = As(t) + n(t) \]
- Data matrix
  \[ X = AS + N \]
- Unknown mixture matrix A
- Unknown signals S
- Input signals are non-Gaussian and statistically independent
Blind Signal Separation (instantaneous)

- Two Stage Technique
- Mixing applies shearing, stretching and rotating
- Decorrelation undoes shearing and stretching
- Hidden rotation remains

\[
\begin{align*}
E[vv^H] &= I \\
QQ^H &= I
\end{align*}
\]  \[\Rightarrow E[Qvv^HQ^H] = QIQ^H = I\]
Second Order Stage (Instantaneous)

- Decorrelate Signals
  \[ V_S = WX \]
  \[ \sum_{t=1}^{T} v_i(t)v_j(t) = \delta_{ij} \]
  \[ V_SV_S^H = I \]

- Use Conventional 2\textsuperscript{nd} order techniques
- For example SVD or PCA
Hidden Rotation Matrix

- By definition

\[ V_s V_s^H = I_s \]

- Now define

\[ \tilde{V}_s = QV_s \]

- Then

\[ \tilde{V}_s \tilde{V}_s^H = QV_s V_s^H Q^H = I_s \]

- Can only conclude that

\[ S = QV_s \]
Convolutive Mixing

- Effects of dispersion, multipath etc
  - Typical of urban environment
  - Cocktail party effect

\[
\begin{align*}
\hat{s}_1(t) & \implies x_1(t) \\
\hat{s}_2(t) & \implies x_2(t)
\end{align*}
\]
Polynomial Matrices

- Convolution is product of z-transforms

\[ x(z) = h(z)s(z) \]

- Two signals and two sensors

\[
\begin{bmatrix}
  x_1(z) \\
  x_2(z)
\end{bmatrix} =
\begin{bmatrix}
  h_{11}(z) & h_{12}(z) \\
  h_{21}(z) & h_{22}(z)
\end{bmatrix}
\begin{bmatrix}
  s_1(z) \\
  s_2(z)
\end{bmatrix}
\]

- Polynomial matrix \( H(z) \)

- Need for new mathematical algorithms
Second Order Stage (Convolutive)

• Strong Decorrelation

\[ \sum_{t=1}^{T} v_i(t)v_j(t - \tau) = \sigma_i(\tau)\delta_{ij} \]

\[ v_i(z)v_j\left(\frac{1}{Z}\right) = \sigma_i(z)\delta_{ij} \]

\[ V(z)V^T\left(\frac{1}{Z}\right) = \begin{bmatrix} \sigma_1(z) & 0 \\ 0 & \sigma_2(z) \end{bmatrix} \]

• Whiten or equalise spectra
Hidden Paraunitary Matrix

- Paraconjugation
  \[ \tilde{H}(z) = H^T \left( \frac{1}{z} \right) \]

- Paraunitary matrix
  \[ H(z)\tilde{H}(z) = \tilde{H}(z)H(z) = I \]

- Apply a decorrelation and whitening filter (2nd order)
  \[ V(z)\tilde{V}(z) = I \]

- Hidden paraunitary matrix
  \[ H(z)V(z)\tilde{V}(z)\tilde{H}(z) = I \]
Vaidyanathans Parameterisation

- Decomposes all paraunitary matrices into a series of rotation interspersed with delays
New Parameterisation

- Elementary Paraunitary Matrices specified by \((N, \theta)\)
- Repeated use of Instantaneous ICA algorithms

\[
\begin{align*}
\alpha + \alpha & \implies Z_N \cos(\theta) \\
\text{Repeat} & \implies \text{Elementary Paraunitary Matrix}
\end{align*}
\]
Simple Example

Original Signals

Mixed Signals
Simple Example-cont

Separated Signals

Recovered Signals

QinetiQ
Comparison with Comon et al

- Published December 2001
- Sufficient information to reproduce trials
- Used BPSK signals, with data lengths of 250 or 400
- Output measure the Bit Error Rate
- Averaged over 500 trials
BER Comparison

- Most have 5-10% BER at 5dB
- At 10dB+ SBR has reduced BER below 1%
- At 15dB+ SBR has reduced BER below 0.2%
- At 10dB+ SBR is better than the reference algorithm by a factor of at least three
Conclusions

- Novel approach to blind convolutive unmixing
- Strong decorrelation
- Temporal whitening / spectral equalisation
- Hidden paraunitary matrix
- Sequence of elementary paraunitary matrices
- Repeated application of instantaneous ICA
- Initial results very encouraging