THE WAVE DIGITAL REED: A PASSIVE FORMULATION

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ABSTRACT

In this short paper, we address the numerical simulation of the single reed excitation mechanism. In particular, we discuss a formalism for approaching the lumped nonlinearity inherent in such a model using a circuit model and the application of *wave digital filters* (WDFs), which are of interest in that they allow simple stability verification, a property which is not generally guaranteed if one employs straightforward numerical methods. We present first a standard reed model, then its circuit representation, then finally the associated wave digital network. We then enter into some implementation issues, such as the solution of nonlinear algebraic equations, and the removal of delay-free loops, and present simulation results.

1. INTRODUCTION

We address here the numerical simulation of the single reed excitation mechanism. Most classical models [1, 2, 4, 5] incorporate a lumped nonlinearity, yet retain an underlying structure which is analogous to that of an electrical circuit. A logical next step is the application of *wave digital filters* (WDFs) [6, 7] as a numerical simulation method, as per [8, 9]. There are several benefits to such an approach: First, the circuit model makes the energetic properties of the system (in particular, the separation into active and passive elements, connected via Kirchhoff's equations) readily apparent. Second, it then becomes straightforward to transfer these energetic properties to discrete time, thus allowing simple stability verification. In other words, the total stored energy of the system may be bounded in terms of the energy supplied by the source (i.e., the mouth pressure). This numerical stability has not been addressed in the context of straightforward finite-difference type approaches such as the K-method [5]. Third, such a wave-based model can then be easily connected to other such models for the instrument body, such as *digital waveguides* [10], as was done in the present case of the reed instrument in [11], though in a simplified form. In Section 2 we present a classical model of the nonlinear reed; then, after a brief summary of wave digital filtering methods in Section 3, we present its circuit representation, and then finally the associated wave digital network in Section 4. In Section 5 we enter into a discussion of some implementation issues, such as the solution of nonlinear algebraic equations and the removal of delay-free loops. Simulation results are presented in Section 6. The approach is similar to that applied to the nonlinearity in a piano hammer excitation [12, 13].

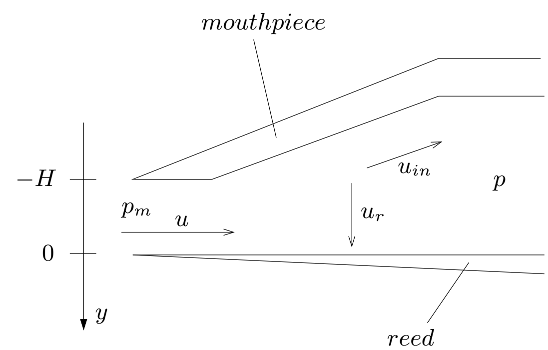


Figure 1: *Schematic depiction of the single reed excitation model.*

2. A SINGLE REED MODEL

Models of the nonlinear reed have appeared frequently in the literature. One of the simpler forms [2, 5], in which the reed is modelled as a lumped harmonic oscillator, driven by a pressure drop across the mouthpiece opening (the slit), is quite standard; we thus state it directly, with reference to Figure 1, which shows the relevant physical quantities.

The reed motion is described by

$$\frac{d^2y}{dt^2} + g\frac{dy}{dt} + \omega^2 y = -\Delta p/\mu \quad (1)$$

Here y is the deflection of the reed from equilibrium (taken positive when the reed is bending outwards, and constrained to be greater than $-H$, where H is the slit width at equilibrium, and

$$\Delta p = p_m - p \quad (2)$$

is the pressure difference between the pressure p_m at the mouthpiece entrance (the blowing pressure) and the pressure p at the interface with the instrument body. μ is the mass per unit area of the reed, g is a loss coefficient, and ω is the resonant frequency of the lumped reed model.

The pressure drop Δp across the mouthpiece is related to the acoustic volume flow u at the input by

$$\Delta p = \frac{\rho}{2}(\frac{u}{w(y+H)})^2\mathrm{sgn}(u) \quad (3)$$

Finally, the acoustic flow u_{in} at the entrance to the instrument body can be written as a difference of the two contributions u (at the entrance to the mouthpiece) and the flow induced by the motion of the reed u_r, or

$$u_{in} = u - u_r \tag{4}$$

u_r is defined by

$$u_r = S_r \frac{dy}{dt} \tag{5}$$

where S_r is an equivalent surface area for the reed.

The equations (1)—(5) form a system of five equations in the six variables Δp, p, u, u_r, u_{in} and y; they are closed by a connection to the instrument body (assumed here to be a cylindrical tube of admittance Y_0). In this simple model, we follow Borin [5] in neglecting the effect of reed-beating (i.e., full closure of the mouthpiece opening); we hope to include this effect in a more complete model, and will make some comments on such an extension in Section 7.

3. BACKGROUND: CIRCUIT ELEMENTS AND WAVE DIGITAL FILTERS

In this section, we summarize the basics of the theory of wave digital filters.

3.1. Circuit Elements and Connections

The circuit elements of which we will make use in this paper, namely the *inductor*, *capacitor* and *resistive voltage source* are as shown in Figure 2. All are defined by some form of relation between a voltage v and a current i, more specifically

$$\text{Inductor}: \quad v = L\frac{di}{dt} \tag{6a}$$

$$\text{Capacitor}: \quad i = C\frac{dv}{dt} \tag{6b}$$

$$\text{Resistive source}: \quad v = e + Ri \tag{6c}$$

where L is an *inductance*, C a *capacitance*, R a *resistance* and $e = e(t)$ is an independent voltage source. For constant $L \geq 0$

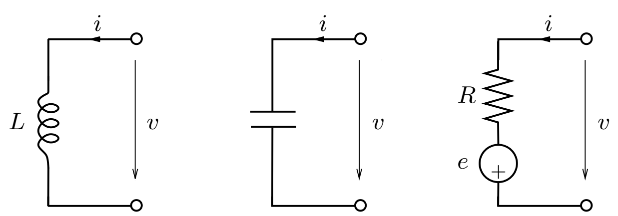


Figure 2: *Left to right: An inductor, of inductance L, a capacitor of capacitance C, and a resistive source, of resistance R.*

and $C \geq 0$, the inductor and capacitor are passive circuit elements [14], and indeed lossless. By this, we mean that the power supplied to either of these elements through its terminals is wholly converted into a stored energy, to be eventually returned to the rest of the network to which it is connected (i.e., it is *reactive*) [14, 15]. For the inductor, for instance, the supplied power is

$$vi = Li\frac{di}{dt} = \frac{d}{dt}(\frac{1}{2}Li^2)$$

so that if the stored energy is defined by $\frac{1}{2}Li^2$, the equation above can be simply read off as ”the power supplied is equal to the rate of change of stored energy.” The non-negativity of L (and also C) is crucial here, in that the energy measure is otherwise not a positive function of the state variables (and cannot be used to bound the size of the state itself, as it evolves). We note that it is possible to extend the definition of either the inductor and capacitor to a non-linear, but nonetheless lossless form (as discussed, in the context of multidimensional wave digital filters in [16], and as applied to the piano hammer nonlinearity in [12, 13]), but we will not make use of such elements in modeling reed vibration.

The resistive voltage source is a series connection of a dependent source (a resistor) and an independent source, and is, in general, non-passive, or *active*. The power supplied to the element is

$$vi = ei + Ri^2$$

In order that at least the dependent part be passive, we require that $R \geq 0$, but notice that in this case, the resistance R may have, in general, any functional dependence whatsoever on time t, or perhaps other state variables without compromising passivity, as long as it remains non-negative.

The circuit elements mentioned above are known as *one-port* elements; they may be connected together (always pairwise) through the application of *Kirchhoff's Laws* [14]. For a *series* connection of N one-ports, of voltages v_k and currents i_k, $k = 1, \ldots, N$, Kirchhoff's Series Law requires that

$$i_1 = i_2 = \ldots = i_N$$
$$v_1 + v_2 + \ldots + v_N = 0$$

For a *parallel* connection, Kirchhoff's Voltage Law requires

$$v_1 = v_2 = \ldots = v_N$$
$$i_1 + i_2 + \ldots + i_N = 0$$

Both laws also enforce passivity (indeed, losslessness) as may be easily verified; in other words, instantaneous power is preserved in such a connection. As a result, any closed network made up of Kirchhoff connections of passive elements must be, as a whole, passive, by Tellegen's Theorem [14].

3.2. Discretization

A wave digital network corresponding to an analog network is always arrived at through discretization of derivatives via the trapezoid rule, which can be written, in operator form, as

$$\frac{d}{dt} \Longrightarrow \frac{2}{T}(1+\delta_t)^{-1}(1-\delta_t)$$

Here, T is the time-step ($1/T$ is the sampling rate) and δ_t is a unit shift operator, which, when applied to a discrete-time sequence y^n, indexed by integer n, gives

$$\delta_t y^n = y^{n-1}$$

(Note that we use a superscripted index to distinguish a discrete-time signal from a continuous-time signal.) Such time series will typically be, in a circuit setting, the discrete-time equivalents of voltages and currents. We note that for linear and time-invariant networks, the trapezoid rule can equally well be interpreted, in the frequency domain as a *bilinear transformation* [6], mapping an analog frequency variable s to a discrete frequency variable z. In the present case of the single reed, the circuit model will be, necessarily, nonlinear, and use of the bilinear transformation interpretation is not permissible.

The equations defining the inductor and capacitor become, under this discretization rule,

$$\text{Inductor}: \quad v^n + v^{n-1} = \frac{2L}{T}(i^n - i^{n-1}) \tag{7a}$$

$$\text{Capacitor}: \quad i^n + i^{n-1} = \frac{2C}{T}(v^n - v^{n-1}) \tag{7b}$$

The equation defining the resistive voltage source, as it does not involve a differential operator, becomes simply

$$\text{Resistive source}: \quad v^n = e^n + R^n i^n \tag{8}$$

Kirchhoff's Laws, like the equation defining a resistive source, hold instantaneously, and thus remain unchanged under discretization.

3.3. Wave Variables and Wave Digital Elements

The next step towards a wave digital realization of an analog circuit network involves the introduction of *wave variables* [6, 7]. For a one-port of voltage v and current i, power-normalized wave variables are defined by

$$\text{Input wave}: \quad a = \frac{v + iR_0}{2\sqrt{R_0}} \tag{9a}$$

$$\text{Output wave}: \quad b = \frac{v - iR_0}{2\sqrt{R_0}} \tag{9b}$$

(We note that in any expression for which the time index n is omitted, it is assumed to hold instantaneously.) This new set of variables is parameterized by R_0, called the port-resistance, which is constrained, in order to ensure passivity, to be non-negative. We note that power-normalized wave variables (as opposed to the more standard voltage wave variables [6]) are essential in any wave digital discretization of a system which is not linear and time-invariant.

Though R_0 is in general a free parameter, when properly set, it can yield discrete-time circuit elements which are of very simple form. In the cases of the inductor and capacitor, the discretized equations (7) become

$$\text{WD inductor}: \quad b^n = -a^{n-1} \qquad R_0 = \frac{2L}{T} \tag{10a}$$

$$\text{WD capacitor}: \quad b^n = a^{n-1} \qquad R_0 = \frac{T}{2C} \tag{10b}$$

for the choices of port-resistance given above. In other words, they can be implemented as shifts, with or without sign-inversion, respectively.

For the resistive source, defined by (8), the wave digital counterpart becomes

$$\text{WD resistive source}: \quad b^n = \frac{e^n}{2\sqrt{R^n}} \qquad R_0^n = R^n \tag{11}$$

Here, note that for a nonlinear or time-varying resistance, the port-resistance will also exhibit the same dependence. Also note that the output of such a source is dependent only on the discrete-time source voltage e^n, and not the input wave a^n. The wave digital inductor, capacitor and resistive source are shown in Figure 3.

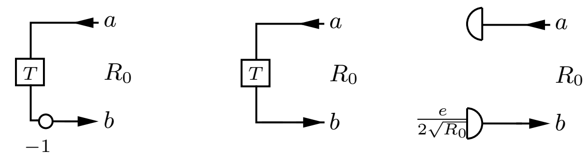


Figure 3: *Left to right: A wave digital inductor, capacitor and resistive source. Values of R_0 are set according to* (10) *and* (11).

3.4. Adaptors

As we mentioned earlier, Kirchhoff's Laws remain unchanged under discretization. For N one-ports, of voltages v_k, and currents i_k, $k = 1, \ldots, N$, we may define wave variables a_k and b_k, with respect to port resistance R_k, as per (9) [1], and Kirchhoff's Laws become the following *scattering equations*:

$$\text{Series}: \quad b_k = a_k - \frac{2\sqrt{R_k}}{\sum_{j=1}^N R_j}\sum_{j=1}^N \sqrt{R_j}a_j \tag{12a}$$

$$\text{Parallel}: \quad b_k = -a_k + \frac{2\sqrt{G_k}}{\sum_{j=1}^N G_j}\sum_{j=1}^N \sqrt{G_j}a_j \tag{12b}$$

where in the case of a parallel connection, we have defined the *port conductance* by

$$G_k = \frac{1}{R_k}$$

These equations may be simplified by defining the *junction current* i (in the series case) and *junction voltage* v (in the parallel case) by

$$\text{Junction current}: \quad i = \frac{2}{\sum_{j=1}^N R_j}\sum_{j=1}^N \sqrt{R_j}a_j \tag{13a}$$

$$\text{Junction voltage}: \quad v = \frac{2}{\sum_{j=1}^N G_j}\sum_{j=1}^N \sqrt{G_j}a_j \tag{13b}$$

in which case the scattering equations become

$$\text{Series}: \quad b_k = a_k - \sqrt{R_k}i \tag{14a}$$

$$\text{Parallel}: \quad b_k = -a_k + \sqrt{G_k}v \tag{14b}$$

The scattering equations above describe what is known, in the WDF literature, as an *adaptor*[18]; graphical representations of general adaptors are shown in Figure 4.

Reflection-free Ports

An important special case occurs when one of the port resistances or conductances of an adaptor remains free (i.e., it is not determined by the properties of a circuit element). Supposing the qth port to be such a port, then if the port resistance R_q or conductance

[1] Note that in keeping with the WDF literature, we will treat a Kirchhoff connection as an N-port in its own right, so that waves a and b refer to the *inputs and outputs of the connection*, which is opposite to the orientation with respect to the one-ports themselves.

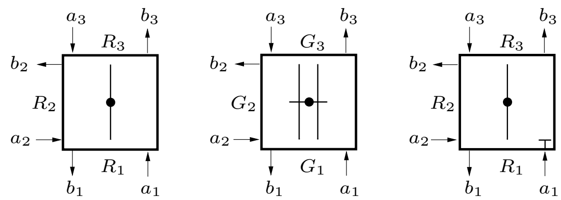


Figure 4: *Left to right: A three-port series adaptor, a three-port parallel adaptor, and a three-port series adaptor for which port 1 is reflection-free.*

G_q is chosen as

$$\text{Series case}: \qquad R_q = \sum_{j=1, j\neq q}^{N} R_j \qquad (15a)$$

$$\text{Parallel case}: \qquad G_q = \sum_{j=1, j\neq q}^{N} G_j \qquad (15b)$$

the scattering equations have the interesting property that at the qth port, the output b_q is independent of the input a_q:

$$\text{Series connection}: \qquad b_q = -\frac{1}{\sqrt{R_q}} \sum_{j=1, j\neq q}^{N} \sqrt{R_j} a_j \qquad (16a)$$

$$\text{Parallel connection}: \qquad b_q = \frac{1}{\sqrt{G_q}} \sum_{j=1, j\neq q}^{N} \sqrt{G_j} a_j \qquad (16b)$$

A graphical representation of a series adaptor with a reflection-free port is shown at right in Figure 4.

Scattering Matrices

It is useful to view the above scattering equations (12) in matrix form. Defining vectors of input and output wave variables by

$$\mathbf{a} = [a_1, \ldots, a_N]^T \qquad \mathbf{b} = [b_1, \ldots, b_N]^T$$

respectively, where ^{T} designates a transposition operation, and the vectors

$$\begin{aligned} \alpha_s &= \sqrt{\frac{2}{\sum_{j=1}^{N} R_j}} [\sqrt{R_1}, \ldots, \sqrt{R_N}]^T \\ \alpha_p &= \sqrt{\frac{2}{\sum_{j=1}^{N} G_j}} [\sqrt{G_1}, \ldots, \sqrt{G_N}]^T \end{aligned}$$

the scattering equations (12) can be written as

$$\text{Series scattering}: \qquad \mathbf{b} = (\mathbf{I} - \alpha_s \alpha_s^T)\mathbf{a} = \mathbf{S}_s \mathbf{a} \qquad (17a)$$

$$\text{Parallel scattering}: \qquad \mathbf{b} = (-\mathbf{I} + \alpha_p \alpha_p^T)\mathbf{a} = \mathbf{S}_p \mathbf{a} \qquad (17b)$$

It is easy to verify that the scattering matrices $\mathbf{S}_s$ and $\mathbf{S}_p$ defined above are orthogonal [19], so that in either the series or parallel case, we must have

$$\mathbf{b}^T \mathbf{b} = \mathbf{a}^T \mathbf{a} \qquad (18)$$

The invariance of the l^2 norm of the set of of wave variables through the scattering operation is a direct result of the losslessness of the Kirchoff connection itself.

An important observation to make at this point is that the port resistances above (which follow from the properties of the elements which make up the Kirchoff connection) may have any functional dependence at all; as long as they remain positive, the scattering operation is orthogonal. A typical problem which arises in nonlinear systems is the case in which one or more port resistances depend on state variables in the system, which are not known *a priori*; this leads to a nonlinear algebraic equation to be solved, as is usual in numerical methods for nonlinear problems. We discuss the implications of this in the present case of the reed model in Section 5.

4. A CIRCUIT MODEL OF THE SINGLE REED AND A WAVE DIGITAL COUNTERPART

Before proceeding directly from the defining equations for the single reed model (i.e., (1) —(5)) to a circuit representation, it is useful to rewrite the system in terms of voltage and current variables. Defining

$$\begin{array}{lll} \Delta v = -\Delta p & v = p & e = p_m \\ i_r = u_r & i = -u & i_{in} = -u_{in} \end{array}$$

and the coefficients

$$\begin{aligned} L &= \frac{\mu}{S_r} \qquad C = \frac{S_r}{\mu\omega^2} \qquad R_1 = \frac{\mu g}{S_r} \\ G_2 &= \sqrt{\frac{2}{\rho|\Delta v|}} w(y+H) \qquad G_0 = Y_0 \end{aligned}$$

the system defining the reed becomes

$$\begin{aligned} v - L\frac{di_r}{dt} - R_1 i_r - \frac{1}{C}\int_0^t i_r(t')dt' - e &= 0 \\ i_{in} - i - i_r &= 0 \\ e + G_2 i &= v \end{aligned}$$

This has an immediate circuit interpretation as shown at left in Figure 5. Notice that there remains an open port, to be connected to the network representing the instrument body (of admittance G_0, corresponding to an acoustic tube). We also note that we have split the source into two separate contributions, each of which is paired with a resistor; thus each source/resistor pair can be treated as a single resistive source. It is possible to arrive at a circuit with only a single source, but the topology becomes marginally more complex.

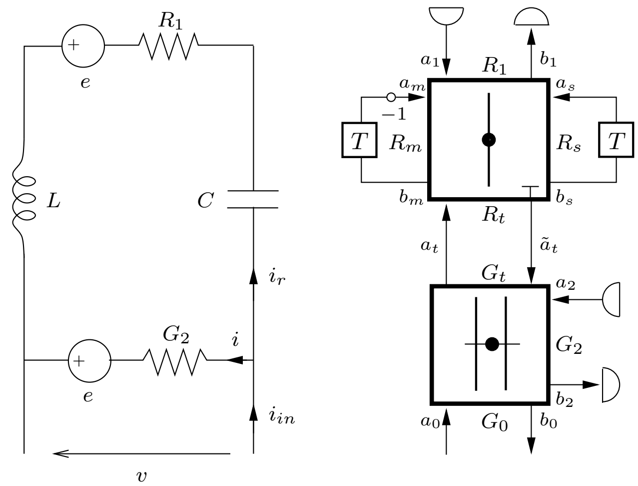
The wave digital counterpart follows directly, and is shown at right in Figure 5. The port resistances corresponding to the inductor and capacitor, as indicated in the figure, are given by

$$R_m = \frac{2L}{T} \qquad R_s = \frac{T}{2C}$$

as per (10). The port resistance and conductance of the resistive sources in the series and parallel adaptor remain as R_1 and G_2, as given in (11).

We thus have a connection between a parallel and a series adaptor; as they are in direct (instantaneous) communication, a delay-free loop results unless we choose one of the two interconnected ports to be reflection-free. This can be done, for the series adaptor, by setting the port resistance, according to (15), as

$$R_t = R_m + R_s + R_1$$



To instrument body — To digital waveguide

Figure 5: *At left, circuit model for the single reed system, and at right its wave digital counterpart.*

The port conductance at the adjoining port at the parallel adaptor is then $G_t = 1/R_t$.

The open port at the parallel adaptor is to be connected to a digital waveguide, of admittance Y_0 (with port conductance $G_0 = Y_0$) [11].

5. IMPLEMENTATION DETAILS

It is worth running through the implementation details for this signal flow graph. All scattering methods (including WDFs and digital waveguides) when programmed, take the form of a recursion, each cycle of which consists of a *scattering* step, and then a *shifting* step. In the first step, all wave variables incident on adaptors are scattered, according to (12). Then, the output values leading to reactive elements such as inductors, capacitors or digital waveguides, are shifted, and new wave variables appear at the inputs to the adaptors.

Examining the WDF diagram of Figure 5, we begin from the series adaptor, and calculate the output $\tilde{a}_t$ at the reflection-free port, which is, from (16),

$$\tilde{a}_t = -\frac{1}{\sqrt{R_t}}(a_m + a_s + a_1)$$

where a_m and a_s are the wave variables approaching from the WD inductor and capacitor, respectively, and $a_1 = -\frac{e}{2\sqrt{R_1}}$ is the wave approaching from the resistive source.

The expression for the junction voltage v at the parallel adaptor can then be written, from (13) in terms of the input wave variables $\tilde{a}_t$ from the reflection-free port, a_0 from the output of the digital waveguide, and $a_2 = \frac{e\sqrt{G_2}}{2}$ as

$$v = \frac{2}{G_t + G_0 + G_2}(\sqrt{G_t}\tilde{a}_t + \sqrt{G_0}a_0 + \sqrt{G_2}a_2)$$

Using the fact that $v = \Delta v + e$, and the definition of a_2 above, this can be rearranged as

$$\Delta v(G_2 + G_t + G_0) + e(G_t + G_0) - 2(\sqrt{G_0}a_0 + \sqrt{G_t}\tilde{a}_t) = 0 \tag{20}$$

Due to the dependence of G_2 on both y and Δv, this equation is the algebraic nonlinearity to be solved. Before doing this, however, we need to find a relationship between y and Δv. This can be obtained as follows:

The voltage across the capacitive port of the series inductor is equal, from (1), to $-\mu\omega^2 y/S_r$, and the junction current is simply i_r. Thus we may write, from the definition of wave variables at the capacitive port and the reflection-free port,

$$\begin{aligned} a_s &= \frac{-\mu\omega^2 y/S_r + R_s i_r}{2\sqrt{R_s}} \\ \tilde{a}_t &= \frac{v - i_r R_t}{2\sqrt{R_t}} = \frac{\Delta v + e - i_r R_t}{2\sqrt{R_t}} \end{aligned}$$

which may be combined to yield the relationship

$$y = \alpha\Delta v + \beta \tag{21}$$

with

$$\begin{aligned} \alpha &= \frac{R_s}{\mu\omega^2 R_t} \\ \beta &= \frac{-R_s(2\sqrt{R_t}\tilde{a}_t - e)/R_t - 2\sqrt{R_s}a_s}{\mu\omega^2} \end{aligned}$$

When the expression (21) is inserted into (20), a pair of cubic equations results, in the variable $q = \sqrt{|\Delta v|}\text{sgn}(\Delta v)$, namely,

$$c_3 q^3 + c_2 q^2 + c_1\text{sgn}(q)q + c_0\text{sgn}(q) \tag{22}$$

with

$$\begin{aligned} c_0 &= -2(\sqrt{G_t}\tilde{a}_t + \sqrt{G_0}a_0) + (G_0 + G_t)e \\ c_1 &= w(H + \beta)\sqrt{2/\rho} \\ c_2 &= G_0 + G_t \\ c_3 &= w\alpha\sqrt{2/\rho} \end{aligned}$$

Once q (and thus Δv) is determined (via an explicit formula or some root-finding method), then so is v, and scattering may be performed at the parallel adaptor. The outputs b_0 (to the digital waveguide) and a_t (to the series adaptor) are then available. Then scattering is performed at the series adaptor, and the outputs b_m and b_s are sent to the WD capacitor and inductor, respectively. The final step is to shift the wave variables stored in the reactive elements (the inductor, capacitor and waveguide).

One fine point regarding the solution of the nonlinear equation (22): though in simulation, there is only one viable solution to the equation which generates a value $y \geq -H$, we were not able to prove that this solution is unique for all choices of the model parameters.

Energy Balance

A perfect power or energy balance can easily be derived, from inspection of the wave digital network in Figure 5. From (18), it is easy to show that, for the wave digital reed coupled to a digital waveguide with perfectly reflecting termination, we have the following power balance:

$$E_s^{n+1} - E_s^n - w_i^n + w_l^n = 0 \tag{23}$$

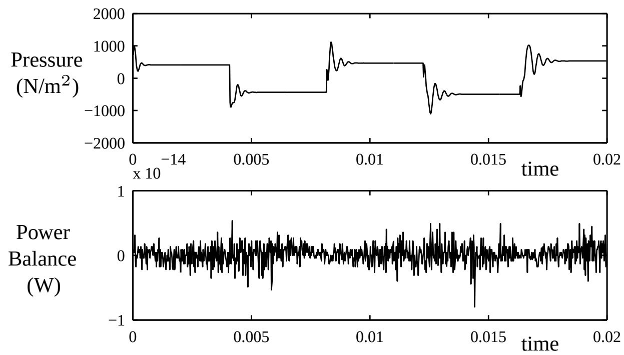


Figure 6: *Top, output pressure p as a function of time for the wave digital reed model. Bottom, the power balance* (23)*.*

where E_s^n is equal to $(a_m^n)^2+(a_s^n)^2$ plus the sum of the squares of the values stored in the digital waveguide, $w_i^n = (a_1^n)^2 + (a_2^n)^2$, and $w_l^n = (b_1^n)^2 + (b_2^n)^2$. E_s^n has the interpretation of a total stored energy in the reed/bore system, w_i^n that of externally-supplied power, and w_l^n that of power loss. In other words, the change in stored energy per sample plus the power lost must equal the power supplied externally. This balance is respected, to machine accuracy, as will be shown in the next section.

6. SIMULATION

For simulation purposes, we choose the model parameters given in [5]. The reed is characterized by $\mu = 0.0231$ kg/m^2, $S_r = 1.46 \cdot 10^{-4}$m^2, $\omega = 23250$ rad/s, $g = 3000$ s^{-1} and $H = 4 \cdot 10^{-4}$m. The mouth pressure p_m is a step function of amplitude 2000 N/m^2, and the bore is an open tube of length 0.7 m, and of acoustic admittance 4.336 $\cdot 10^{-7}$ m^{4}s/kg. The sample rate is chosen as 44 100 Hz. Output pressure, as well as the power balance (23) (which is purely round-off error) are shown in Figure 6.

7. CONCLUSIONS

We have discussed, in this paper, the means by which circuit theory and wave digital filtering principles may be applied to the numerical simulation of single reed model which incorporates a high degree of nonlinearity; as evidenced by simulation results, there is a perfect preservation of a discrete-time energy balance which mirrors that of the continuous-time model.

We have only, however, examined a simplified case of a more complete model. The inertial effects of air mass in the mouthpiece [1] contribute another inductor to the circuit of Figure 5. Extensions to this method could equally well be made for the case of reed beating against the mouthpiece at full closure [2] (i.e., for $y = -H$), for which a wave digital ideal diode [7] (connected across the capacitor) is an appropriate passive nonlinear device. The possibility of variation in H (as another control parameter) is also possible, but will necessarily invoke the use of another source, as such variation necessarily introduces energy (albeit very little) into the system.

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