SIMULATION OF SOUND SOURCE MOTION BY TIME-FREQUENCY FILTERING

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ABSTRACT

The generalisation of conventional linear time invariant filter theory from one dimension to the two-dimensional time-frequency (TF) domain provides a powerful tool for the simulation of complex time variable systems. TF filtering is performed as convolution in the TF domain and is based on non-parametric modelling using direct convolution along the time or frequency axis. A method based on short-time Fourier analysis has been developed to produce non-stationary signals with desirable time and frequency characteristics. This method is faster than non-recursive realisations and yields a simple synthesis procedure. The application of filtering in the TF domain for the simulation of sound source motion is presented.

1. INTRODUCTION

The generalisation of conventional linear time invariant (LTI) filter theory from one dimension to the two-dimensional time-frequency (TF) domain provides a powerful tool for the simulation of complex time variable systems. This type of filtering provides also increased flexibility for detailed filtering operations that cannot be realised with one-dimensional methods. In this manner, considering the output of a time varying system, the interaction between the input signal and the system can be regarded as an operation in the TF domain between the TF expansion of the signal and the TF response of the system [1].

In this paper we continue to develop our recent work, where we discussed the equivalence of TF convolution filtering with respect to the time variable and traditional LTI filtering in the time domain and applied TF filtering for the realisation of time varying and frequency dependent artificial reverberation [2]. Here the equivalence between the traditional frequency modulation and the TF convolution with respect to the frequency variable is studied, and the TF convolution along the frequency axis is employed for the simulation of sound source motion. TF filtering comprises three stages, namely the analysis, the processing and the synthesis stage, and hence is computationally expensive. In order to speed up the calculations a recursive algorithm for the evaluation of the STFT is utilised. This algorithm is faster when compared to non-recursive realisations and yields a simple synthesis procedure.

2. TIME-FREQUENCY FILTERING

The idea of LTI filtering is generalised to filtering operations in the TF plane where synthesis of signals from modified TF distributions yields classes of TF filters in two dimensions [3] [4]. TF masking filters are able to perform localised selection of TF components while TF convolution filters perform a transformation rather than a simple masking in the TF plane.

A schematic presentation of the basic elements of filtering in the TF plane is illustrated in Figure 1. The signals are expanded into the TF domain, where they are filtered by a one- or two-dimensional operation, and the product is synthesised in the time domain using the corresponding synthesis formulae. The TF expansion $S_x(t,\omega)$ of a signal $x(t)$ is convolved with the two-dimensional transfer function $H(t,\omega)$ to yield the expansion $S_y(t,\omega)$ of the response $y(t)$. Since it is desirable to reconstruct the time history of the output after the filtering operation the result of the operation in the TF plane should be a valid TF distribution, i.e. there exists a signal which corresponds to this TF distribution. If this is not the case the output signal can be synthesised in the time domain using approximation techniques [5].

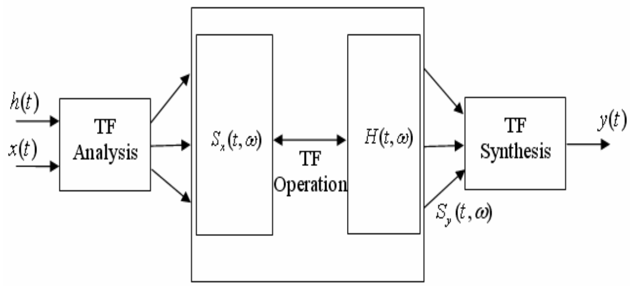

Figure 1: *Filtering operation in the TF plane.*

2.1. TF Convolution along the frequency axis

TF convolution with respect to the frequency variable ω corresponds to multiplication in the time domain if the TF distributions used for this operation preserve the property of multiplication [6]. TF convolution along the frequency axis can be described as

$$S_y(t,\omega) = \int_{-\infty}^{\infty} H(t,\omega-\omega')S_x(t,\omega')d\omega', \qquad (1)$$

where the time histories of the input signal and the system's impulse response are related in the time domain by the product $y(t) = h(t)x(t)$ and in the frequency domain by the convolution $Y(\omega) = H(\omega) * X(\omega)$.

Thus TF convolution along the frequency axis can be implemented as filtering of the time-frequency decomposition of the input signal $x(t)$ for every time instant t_i, as shown in Figure 2. If the TF distribution used for the operation satisfies the property of multiplication and the distributions involved in the operation are valid, then the product of the TF operation is also valid.

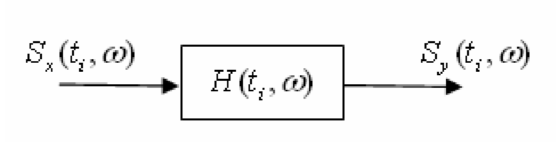

Figure 2: *TF convolution along the frequency axis.*

3. IMPLEMENTATION OF TIME-FREQUENCY FILTERS

TF filtering operation is computationally expensive since it requires the evaluation of the decompositions of the input signal, the system response and the inverse transform of the resulting distribution. In order to accelerate the computation a recursive algorithm for the evaluation of the STFT is employed. The particular algorithm is faster compared to non-recursive realisations, is well suited for real-time applications and yields a simple synthesis procedure.

3.1. Recursive Implementation of the STFT

Many different proposals have appeared in the literature concerning the recursive evaluation of TF distributions [7] [8] [9]. The recursive implementation of the STFT used in this work is based on the notion of the running z-transform that is defined as the short time z-transform of a delayed signal [10]. For a sequence $x(n)$, the running z-transform is

$$\Phi(n,z) = \sum_{k=0}^{N-1} x(n-k) z^{-k}. \tag{2}$$

For a fixed n, $\Phi(n,z)$ is the z-transform in the variable k of the segment $x(n-k)$, $0 \le k \le N-1$ of $x(n)$. The inversion formula, considering evaluation on the unit circle, is

$$x(n) = \frac{1}{N} \sum_{m=0}^{N-1} \Phi(n, w^{-m}), \tag{3}$$

where $w = e^{j\frac{2\pi}{N}}$. Using this definition it is easy to recognise in the running z-transform the sampled version of the Fourier transform of a delayed sequence $x(n-k)$, $0 \le k \le N-1$. For simplicity the above formulation assumes a rectangular window function applied to the signal. By substituting $k+1 = p$ in (2) one obtains

$$\Phi(n-1,z) = z \sum_{p=1}^{N} x(n-p) z^{-p} \tag{4}$$

and it follows that the function $\Phi(n, w^{-m})$ satisfies the first order recursion equation

$$\Phi(n,z) - z^{-1}\Phi(n-1,z) = x(n) - z^{-N} x(n-N). \tag{5}$$

By evaluating the running z-transform on the unit circle, i.e. $z = w^{-m}$ the function $\Phi(n, w^{-m})$ has the simple recursive form

$$\Phi(n, w^{-m}) - w^{m}\Phi(n-1, w^{-m}) = x(n) - x(n-N). \tag{6}$$

This defines a discrete recursive system with input $x(n)$, output $\Phi(n, w^{-m})$ and system function

$$S(m,z) = \frac{1 - z^{-N}}{1 - w^{m} z^{-1}}. \tag{7}$$

The system consists of a shift register with output $x(n-N)$, a delay element and a multiplier, as illustrated in Figure 3. Connecting N such systems together in parallel results in a running Discrete Fourier Series (DFS) spectrum analyser that has a filter bank structure, as shown in Figure 4. Prior to reconstructing the signal from its Fourier series expansion, it is possible to filter the TF coefficients according to a time variable system function implementing non-stationary TF convolution filtering.

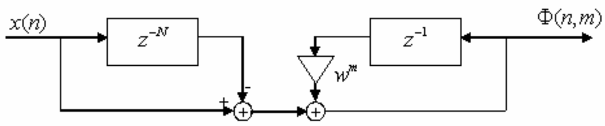

Figure 3: *Elementary filter structure.*

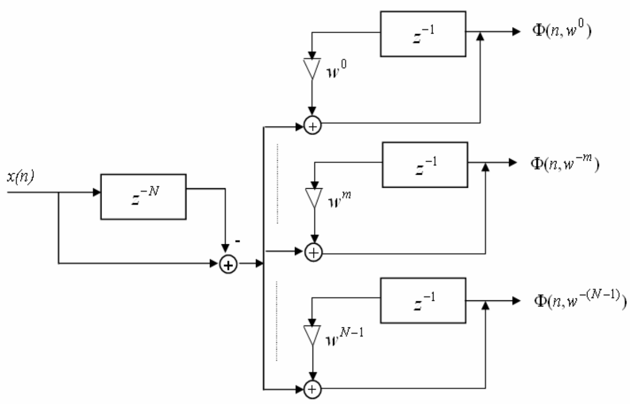

Figure 4: *Recursive DFS analyser.*

4. SIMULATION RESULTS

The Doppler effect is associated with the shift in frequency and wavelength of waves that results from the relative motion between a sound source and a receiver. Thus reproduction of sound source motion derives from controlling frequency characteristics with time. In the context of TF filtering the Doppler effect can be implemented by TF convolution along the frequency axis. A signal whose frequency content undergoes a Doppler shift is expressed as

$$x_D(t) = x(t)e^{\varphi(t,\omega)}, \tag{8}$$

where $\varphi(t,\omega)$ is complex. Considering for example $\varphi(t,\omega) = ipt$ then from (8) it is evident that the frequency content of the signal $x(t)$ has been shifted by p rad/s. The advantage of employing TF filtering is that $\varphi(t,\omega)$ is a function of both time and frequency and thus the frequency shift can be time dependent.

For a discrete implementation of the TF convolution along the frequency axis the infinite integration of (1) has to be replaced by a finite summation. Thus the discrete realisation of the TF filtering along the frequency axis is expressed as

$$S_y(n,m) = \sum_{m'=0}^{M-1} S_h(n,m-m')S_x(n,m'), \tag{9}$$

where S_y, S_x represent the TF expansion of the output and input signal respectively and S_h represents the two dimensional transfer function of the system. The size of the discrete TF matrices may be arbitrary concerning the frequency axis and hence the product will be of size $2M-1$ where M is the frequency resolution of the input signal and the system's transfer function. It is noted that the length of the result is not the same as the resolution of the convolved decompositions. For a valid operation the resolution of the system's response TF decomposition has to be the same as the resolution of the input decomposition.

As an application case a sinusoidal two component signal is filtered according to (9) by a system that corresponds to a variable filter whose frequency response is higher as time increases. The TF decompositions of the original signal and the system's response are obtained using the running DFS analyser and are illustrated in Figure 5 and 6 respectively. The response of the system was designed by specifying its instantaneous frequency to follow the function shown in Figure 7.

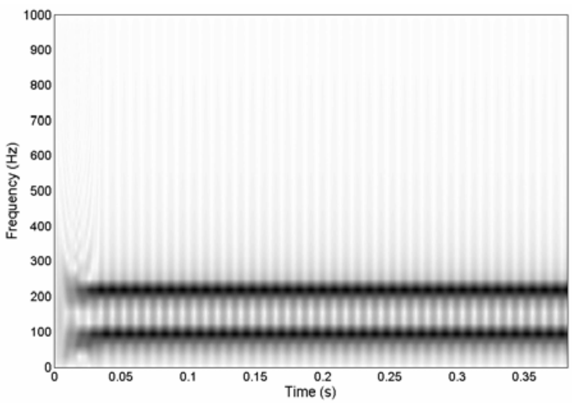

Figure 5: *TF expansion of input signal.*

For the implementation of TF convolution along the frequency axis in case of real signals the direction of convolution has to be considered since repositioning of the frequency energy may destroy the symmetric structure of the spectrum. Thus the TF convolution is performed first up to the folding frequency and then in the opposite direction from the sampling frequency to the folding frequency. For the present example however the analytic representations of the signals were used and thus the operation was only applied in a single direction up to the folding frequency. In order to study the equivalence of TF filtering along the frequency axis and traditional modulation the result of the TF filtering is transformed to the time domain and it is compared with the time series obtained from the modulation operation in the time domain. For the shake of clarity in Figure 8 the detailed comparison between segments of the two time series is presented. It is interesting to observe that the signal corresponding to TF filtering follows the signal corresponding to conventional frequency modulation. The small difference in the amplitude of the two signals is due to the discretisation of the time and frequency variables. The spectral content of the two signals is the same as can be seen from their TF representations shown in Figure 9 and 10 respectively. The fuzzy structure of the initial part of the TF representations is due to the recursive structure of the algorithm for the evaluation of the STFT.

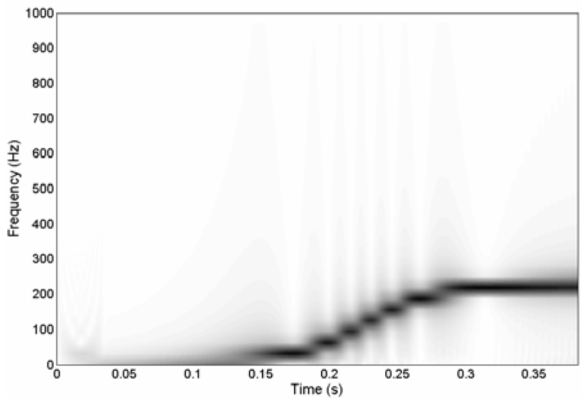

Figure 6: *TF expansion of system's response.*

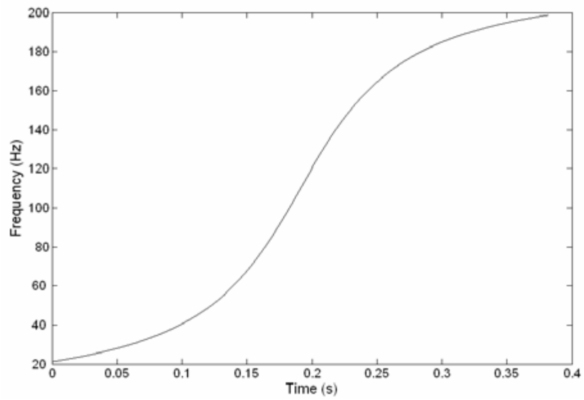

Figure 7: *Instantaneous frequency of system's response.*

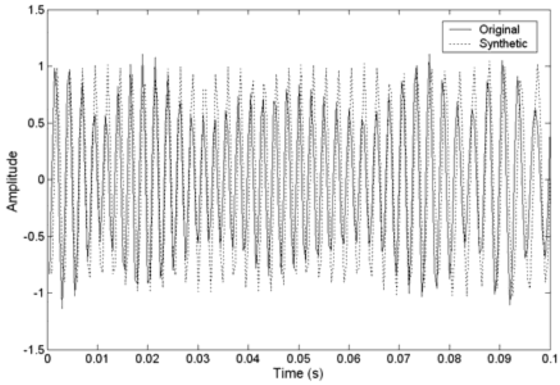

Figure 8: *Detailed comparison between the results of TF filtering along the frequency axis and modulation.*

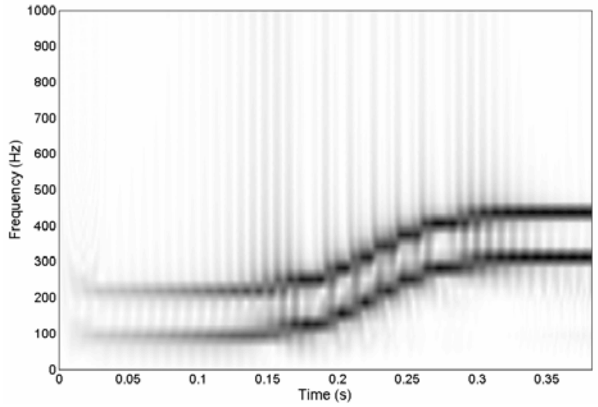

Figure 9: *TF convolution along the frequency axis.*

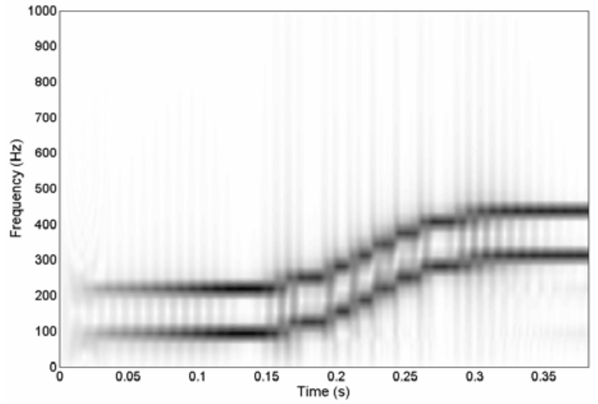

Figure 10: *TF expansion of the result of conventional frequency modulation.*

5. CONCLUSIONS

Filtering of signals in the TF domain provides a tool for detailed filtering operations that cannot be realised with one-dimensional methods. In this work the operation of TF convolution along the frequency axis was considered and was shown to correspond to frequency modulation. However in order to use the advantages of TF filtering it is desirable to synthesise time histories from modified distributions, which requires considerable computation when compared to one-dimensional filtering methods. In order to release this restriction a recursive algorithm for the evaluation of the STFT was applied. The particular algorithm is faster compared to non-recursive realizations, yields a simple synthesis procedure and is well suited for real-time applications. The operation of TF convolution along the frequency axis for the simulation of sound source motion was presented as an application case.

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