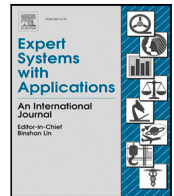




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A Gaussian Process model for UAV localization using millimetre wave radar

José A. Paredes^{a,*}, Fernando J. Álvarez^a, Miles Hansard^b, Khalid Z. Rajab^{b,c}

^a Sensory System Research Group (GISS), University of Extremadura, Av. de Elvas s/n, 06006 Badajoz, Spain

^b Centre for Advanced Robotics (ARQ), School of EECS, Queen Mary University of London, Mile End Road, London E1 4NS, UK

^c NodeNs Medical Ltd., 10 Bloomsbury Way, London WC1A 2SL, UK

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ABSTRACT

The detection and positioning of unmanned aerial vehicles has become essential for both automation and surveillance tasks, in recent years. The design of accurate drone localization systems is challenging, especially in cluttered environments, where the target may be partially or even completely obscured. This paper proposes a precise detection and 3D localization system for drones, by means of a millimetre wave radar. Drone locations are estimated from spatial heatmaps of the received radar signals, which are obtained by applying the super-resolution MUSIC algorithm. These estimates are improved by analysis of the micro-Doppler effect, generated by the rotating propellers, which aids detection in poor visibility conditions. A novel Gaussian Process Regression model is developed, in order to compensate for systematic biases in the radar data. The complete system produces accurate estimates of the target range and direction, and is shown to outperform direct spectral analysis methods.

1. Introduction

The development of reliable drone detection systems has become important, owing to the routine use of unmanned aerial vehicles (UAVs) in daily life. In recent years, these devices have been used for surveillance (Boddhu et al., 2013), logistical (Kellermann et al., 2020), audio-visual (Harvard et al., 2020) and military (de Swarte et al., 2019) tasks. All of these applications require a positioning system of some kind, which provides input to the process for both detection and navigation purposes. However, most of the research on this topic is not primarily concerned with establishing the precise absolute position of the drone with respect to a previously known location, but rather with general detection and tracking. For instance, one of the greatest concerns so far has had to do with the security risks posed by these devices, as discussed in Samland et al. (2012). In particular, much research has focused on detecting and tracking potentially hostile devices, as described in Guvenç et al. (2018, 2017) where the authors present a system consisting of a ground radar and a monitoring drone fleet with the final objective of interdicting malicious UAVs.

Another well-known goal is autonomous guidance, given that the device may travel beyond the communication range of the operator (Kendoul, 2012). In this regard, the basic problem of global positioning has already been solved by the inclusion of a Global Navigation Satellite System (GNSS) signal receiver for the most advanced drones, as can be seen in the features of several current off-the-shelf devices,

such as Phantom (DJI, 2020), or in works such as Tahar and Kamarudin (2016), in which some corrections are applied to the data delivered by the GNSS chip, based on a prior statistical study of the noise and bias in these measurements.

Localization is still problematic in GNSS-denied environments, and in local contexts where an accurate position could be mandatory. The applications developed for these purposes can be tackled either in a dynamic way, as presented above, where other drones carrying different sensors provide assistance to the central system, or directly from the ground, i.e. from a known fixed location, in which the sensor is installed.

The second strategy requires neither an extensive infrastructure, nor the calibration of dynamic reference systems. Therefore, this paper proposes an accurate detection and 3D localization system for drones, designed for cluttered environments, using only a single commodity millimetre wave (mmWave) radar, in a fixed location. The general setup is illustrated in Fig. 1. The device uses short-wavelength electromagnetic signals, which enables detection through a variety of materials, even in optically challenging environmental conditions, such as darkness, fog, or smoke. These sensors are lightweight, and can easily be mounted inside a plastic car bumper, a security camera housing, or a robotic or drone housing (Ferguson et al., 2018). In general, mmWave technology permits the use of very small antennas, enabling

* Corresponding author.

E-mail addresses: japaredesm@unex.es (J.A. Paredes), fafranco@unex.es (F.J. Álvarez), miles.hansard@qmul.ac.uk (M. Hansard), khalid@nodens.eu (K.Z. Rajab).

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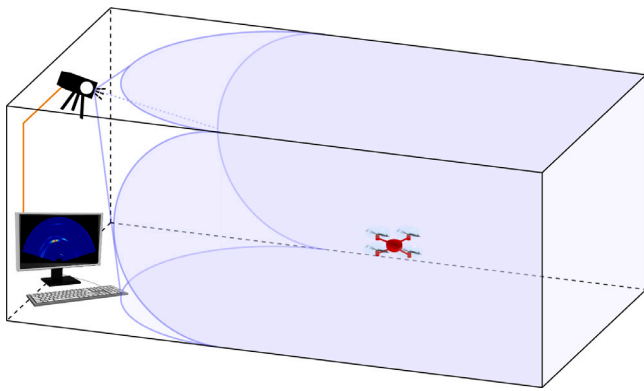


Fig. 1. Overview of the proposed system. The radar is positioned in order to cover as much flying space as possible. In this example, it is placed near the ceiling and slightly tilted towards the ground, in order to increase the RCS (radar cross-section) of the drone.

integration within the sensing unit, and greater system miniaturization. These characteristics make mmWave technology appropriate for use in constrained environments, such as warehouses or urban delivery areas (Alwateer & Loke, 2020).

The contributions of this paper are as follows. A new way to enhance the detection of drones in mmWave radar data is presented in Section 3.1. Section 3.2 develops new univariate and multivariate Gaussian process (GP) regression models, for accurate range and direction estimation. Section 3.3 proposes a spline-based dimension reduction method, to be used in conjunction with the GP regression. A combined mmWave radar and camera system, which allows ground-truth optical measurements to be recorded, is described in Section 4.1. Our basic localization algorithms are evaluated in Section 4.2. The more sophisticated GP regression models are evaluated in Section 4.3. We report the final configuration of an effective UAV system, based on our experiments.

The organization of this paper is as follows. The relevant drone detection and localization systems are reviewed in Section 2. The Appendix describes the main characteristics of the mmWave radar system, and the initial signal processing tasks. The proposed positioning algorithm is developed in Section 3. The complete system is evaluated, and the results are analysed, in Section 4.

2. Drone localization methods

A wide range of different technologies have been applied for detection and localization of drones. This section presents a brief review of the topic, including acoustic, optical, and radar modalities.

2.1. Acoustic methods

The first technology to be considered is that of acoustic analysis. Several works have specialized in detection of the characteristic propeller ‘buzz’ (Benyamin & Goldman, 2014; Busset et al., 2015). However, the applicability of this approach is limited by the cost of a complex microphone matrix, and by its high susceptibility to adverse weather conditions (e.g. wind or rain). Other solutions involve the fusion of different sensors, as proposed in Paredes et al. (2017), where an ultrasonic device estimates 2D position and a time of flight (ToF) camera provides the flying altitude.

2.2. Optical methods

Optical systems use one or more cameras to detect drones, based on methods from computer vision. This approach is one of the oldest, as described in Amidi et al. (1999), in which a camera looking

at the ground is mounted in a drone. The position is estimated by detecting known visual markers. Somewhat later, new marker-based systems emerged, for use with micro aerial vehicles (MAVs) (Boudjit & Larbes, 2015; Jiménez Lugo & Zell, 2014; Masselli & Zell, 2012; Santana et al., 2014; Vidal et al., 2017). A more recent work presents an accurate detection and tracking system for MAVs by means of a single ToF camera (Paredes et al., 2020). Nonetheless, while camera-based systems produce high resolution images across a wide field-of-view (FOV), detection relies on measuring reflected light. This makes them very susceptible to occlusions (by objects crossing the line of sight) and to scattering (by airborne particles such as fog or smoke).

2.3. Radar methods

Radar systems have been used to avoid the drawbacks of acoustic and optical technologies. The main difference from the former is in the signal that is used: radio frequency (RF) waves. Due to its wavelength and beam spread, a radar can return multiple readings from the same transmission, and generate a spatial representation of the environment. By capturing the reflected signal, a radar system can determine range, velocity and direction. An example can be found in Multerer et al. (2017), in which the authors propose the use of a 3D-type radar to track drones. To differentiate between dangerous and non-dangerous targets, they have their movements evaluated in real time and, if the algorithm determines it, a directional antenna generates a WiFi signal that jams the control of the device.

As mmWave-hardware has improved, more complex tasks have become feasible for these devices. The operation of a 2D detection and tracking system for UAVs, using mmWave, is explored in de Haag et al. (2016), where a comparison with a light detection and ranging (LiDAR) system can be found. The authors are able to detect drones at a distance up to 25m, although clutter at less than 5m prevents a proper detection. Besides, when a UAV is intended to be detected at low grazing angles, the multipath effect due to land clutter could mask the direct reflections from the device, as explained in Ezuma et al. (2019). A detailed model of this effect is presented in this work, where the simulations show how the optimal detection performance of mmWave radar depends on the RCS of the targeted MAV, the radar properties, and the properties of the land clutter. Other works, including Dogru et al. (2019) and Dogru and Marques (2020), focus on pursuing one drone with another one. The authors make use of a 2D radar, mounted in the follower, in order to detect the target drone. The missing axis information is extracted through geometric calculations, by estimating the first drone position.

In order to detect and identify UAVs via a radar device, it is often useful to take into account the micro-Doppler effect. A radar target will exhibit a Doppler frequency shift associated with its bulk motion (i.e. the velocity associated with a drone’s flight path), but will also experience micro-motion dynamics – such as mechanical vibrations or rotations – which induce micro-Doppler modulations on the returned signal (Chen et al., 2006). The unique micro-Doppler signatures of a radar target can provide characteristic information about its properties and actions. This has been used to improve drone detection, such as in Caris et al. (2016) where two possible approaches for perimeter surveillance are presented. They cover distances from 10m to several hundreds metres, and they assert that the velocity (modulus and direction) can be estimated by analysing the micro-Doppler effect. This effect has been also applied for classification of drones (Fioranelli et al., 2015; Jian et al., 2017; Nanzer & Chen, 2017; Sun et al., 2018; Tahmoush, 2015), because the Doppler signature can be considered unique for every target, and to discriminate them from other potential flying targets, like birds, owing to the propeller rotation signal (Ezuma et al., 2019; Rahman & Robertson, 2018).

None of these works have addressed a precise drone detection and positioning in cluttered environments, where the clutter (including other non-drone targets) can make the detection unreliable or even impossible. These considerations are important, owing to the increasing

use of drones in complex environments, such as warehouses and urban delivery areas. Hence, this paper proposes an accurate 3D localization system for UAVs in these scenarios, based on a novel approach that applies Gaussian Process Regression (GPR) to the radar information, after improving target detection via the micro-Doppler effect generated by the propellers. Wang et al. (2018) took a related approach to *direction of arrival* estimation, from radar data, using Support Vector Regression (SVR). Our perspective is similar, but we obtain full 3D localization, and we use GPR rather than SVR. We argue that the GPR method has significant advantages, because it provides a probabilistic model of the data, including confidence intervals for prediction.

3. Proposed localization algorithm

The proposed mmWave radar localization system uses the following data, which is provided by the device: a 3D range-frame array \mathbf{Q} , in conjunction with a 2D range-Doppler array \mathbf{Q}_D . A non-negative 3D array \mathbf{P} , representing the signal intensity at any point $[r, \theta, \phi]$, can be obtained directly from the magnitude $|\text{FFT}(\mathbf{Q})|$. In this work, however, we use the multiple signal classification (MUSIC) algorithm, which involves the appropriate steering matrix \mathbf{A} , for the antenna layout (see Section 4.1). Hence the intensity array is computed as

$$\mathbf{P} \leftarrow |\text{MUSIC}(\mathbf{Q}, \mathbf{A})|. \quad (1)$$

The basic principles of radar signal processing (including FFT and MUSIC methods) are summarized in the Appendix.

If one of the angular coordinates is fixed, then the resulting 2D slice through \mathbf{P} corresponds to plane through the centre of the antenna, as indicated in Fig. 2. In the case of the Doppler array, it suffices to compute the absolute value, in order to obtain a range-velocity heatmap:

$$\mathbf{P}_D \leftarrow |\mathbf{Q}_D| \quad (2)$$

The next step in the proposed localization algorithm is to perform pointwise background subtraction

$$\mathbf{P}' = \mathbf{P} - \text{E}[\mathbf{P}]_S \quad (3)$$

$$\mathbf{P}'_D = \mathbf{P}_D - \text{E}[\mathbf{P}_D]_S \quad (4)$$

where \mathbf{P}' and \mathbf{P}'_D are the signals relative to the corresponding temporal averages, denoted by $\text{E}[\cdot]_S$. The latter are taken over a reference time interval $S = [s_0, s_1]$, during which the target is absent, and the scene is static. This step reduces the effects of environmental clutter, by removing the background component from the signal.

For clarity of exposition, we now regard the 3D array \mathbf{P}' and the 2D array \mathbf{P}'_D , from (3) and (4), as non-negative scalar functions $P'(r, \theta, \phi)$ and $P'_D(r, v)$, respectively. This notation also allows us to consider, for example, bilinear interpolation of the data at non-integer coordinates (although we do not require it here). The 2D azimuth and elevation maps $P'_A(r, \theta)$ and $P'_E(r, \phi)$, in the analogous notation, can be obtained by taking the maximum over one angular dimension, in each case:

$$P'_A(r, \theta) = \max_{\phi} P'(r, \theta, \phi), \quad \phi \in [\phi_{\min}, \phi_{\max}], \quad (5)$$

$$P'_E(r, \phi) = \max_{\theta} P'(r, \theta, \phi), \quad \theta \in [\theta_{\min}, \theta_{\max}]. \quad (6)$$

The resulting 2D maps are separately normalized, by the maximum value, in each case. A similar procedure is used to obtain a 1D function of the scene range:

$$P'(r) = \begin{cases} \max_{\theta} P'_A(r, \theta), & \theta \in [\theta_{\min}, \theta_{\max}], \\ \max_{\phi} P'_E(r, \phi), & \phi \in [\phi_{\min}, \phi_{\max}]. \end{cases} \quad (7)$$

Note that the two possibilities are equivalent: in both cases, $P'(r)$ is the maximum over the same spherical wedge of directions (the only difference is the ordering of the angular max operations).

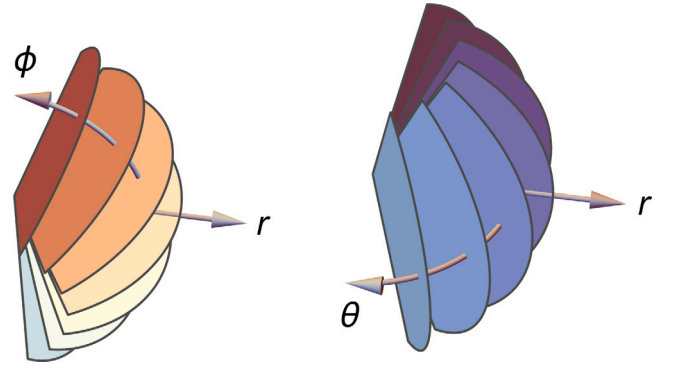


Fig. 2. Left: elevation planes through the original data array $P(r, \theta, \phi)$. A single azimuth map $P'_A(r, \theta)$ is obtained by maximizing over ϕ , for each pair of coordinates (r, θ) , as indicated (5). Right: analogous azimuthal planes. A single elevation map $P'_E(r, \phi)$ is obtained by maximizing over θ , for each pair of coordinates (r, ϕ) , as indicated (6). The angular ranges correspond to the antenna array limits of $\pm 60^\circ$, as specified by the radiation pattern's main beam (while the coarse angular sampling is for illustration only).

3.1. Direct maximum estimation

The actual range of the target can be tentatively estimated from the maximum value of $P'(r)$. However, the detection can be strengthened by considering the \mathbf{P}_D heatmap. When, in addition to the constant Doppler frequency shift induced by the bulk motion of a radar target, the target or any structure on the target undergoes micro-motion dynamics, such as mechanical vibrations or rotations, the micro-motion dynamics induce Doppler modulations on the returned signal, known as micro-Doppler effects (Chen et al., 2006). In the case in question here, the range profile can be also given by:

$$P'_D(r) = \text{mean}_v P'_D(r, v), \quad v \in [v_{\min}, v_{\max}]. \quad (8)$$

Then, the range coordinate r_+ is obtained as:

$$r_+ = \begin{cases} \arg \max_r P'_D(r) & \text{if } \max_r P'_D(r) > I, \\ \arg \max_r P'(r) & \text{otherwise,} \end{cases} \quad (9)$$

where I is a threshold just to verify that this particular Doppler effect is strong enough for the peak to be considered as a possible detection. If the range dimension is reduced as follows

$$P'_D(v) = \max_r P'_D(r, v), \quad r \in [r_{\min}, r_{\max}], \quad (10)$$

then the radial velocity v_+ can be estimated according to

$$v_+ = \arg \max_v P'_D(v) \quad (11)$$

assuming that the target velocity is that of the main intensity peak in this dimension.

Recall, from Eq. (9), that r_+ is the estimated range of the target. Two 1D angular profiles can now be taken at this range, in order to estimate the direction of the target. Indeed the maximum values of these profiles can be used as simple estimates $[\theta_+, \phi_+]$ of the true target direction $[\theta, \phi]$:

$$\theta_+ = \arg \max_{\theta} P'_A(r_+, \theta) \quad (12)$$

$$\phi_+ = \arg \max_{\phi} P'_E(r_+, \phi) \quad (13)$$

However, in order to achieve higher accuracy, a GP regression will be applied to the profiles defined in Eqs. (5) and (6). The resulting system learns to understand the radar data, based on a set of training examples.

3.2. Gaussian process regression

The previous section described a simple target localization method, based on direct peak extraction, in the background-subtracted response signals. In practice, the direct range estimate is relatively accurate, as can be deduced from Eq. (A.2), where a resolution of 3.75 cm is achieved for a typical bandwidth of 4GHz. The angular maxima, however, are much less reliable, as extracted from Eq. (A.6), where can be seen how that relationship is non-linear, so that the greater the angle, the worse the resolution. We therefore propose an alternative localization method, in which the entire angular profiles, at the estimated range, are used as inputs to a supervised learning algorithm.

Specifically, we obtain azimuth and elevation estimates, by analogy with Eqs. (9) (12) and (13), from three GP regressions, each taking the concatenated 1D profiles $[P'(r), P'_A(r_+, \theta), P'_E(r_+, \phi)]$ as inputs. In general, the GP kernel representation may be more effective if the dimensionality of the problem is reduced somewhat. This is possible here, because the data have limited resolution in range (A.2) and azimuth/elevation (A.6). In particular, it will be shown that the azimuth and elevation profiles $[P'_A(r_+, \theta), P'_E(r_+, \phi)]$ can be compactly represented by their coefficients $[\alpha, \epsilon]$ with respect to a fixed spatial basis. We use a standard B-spline representation, as described in Section 3.3. This leads to the following GP regression estimates, by analogy with (12) and (13):

$$\theta_* \leftarrow \text{GPR}_\theta(r_+, \alpha, \epsilon) \quad (14)$$

$$\phi_* \leftarrow \text{GPR}_\phi(r_+, \alpha, \epsilon). \quad (15)$$

We hypothesize that the GPR estimates $[\theta_*, \phi_*]$ are significantly better than the direct maximum estimates $[\theta_+, \phi_+]$ in (12) and (13), for two reasons. Firstly, the relationship between the target direction and the angular data is allowed to be more complex: for example, the peak may be asymmetric, or multimodal. Secondly, the estimator is allowed to be directionally variable, in both dimensions: for example, it is possible to compensate for attenuation, as the true direction $[\theta, \phi]$ approaches the angular limits of the sensor.

The GPR method has the following advantages, in this context. Firstly, it produces sensible estimates outside the range of the available data. Secondly, the model has a clear probabilistic interpretation, including confidence intervals for prediction. Thirdly, the parameterization is physically sensible, based on assumed smoothness of the signal distortions. Finally, the estimation process is exact, and very efficient in this application. We now consider the computational form of the estimates (14) and (15), in the standard GP framework (Rasmussen & Williams, 2006).

Let $\xi = [r_+, \alpha, \epsilon]$ be the representation of a radar reading, from which we wish to obtain a direction estimate $[\theta_*, \phi_*]$, as in (14) and (15). A suitable GP model can be estimated from $i = 1, \dots, L$ training locations of the target, with known azimuths and elevations θ_i and ϕ_i . The corresponding RF measurements are concatenated in the L training vectors ξ_i . The affinity of two such readings can be represented by the exponentiated quadratic kernel:

$$k(\xi_i, \xi_j) = \gamma^2 \exp\left(-\frac{1}{2} d^2(\xi_i, \xi_j)\right) \quad \text{where} \quad (16)$$

$$d^2(\xi_i, \xi_j) = [\xi_i - \xi_j]^T \begin{bmatrix} \gamma_r^2 & & \\ & \gamma_\theta^2 & \\ & & \gamma_\phi^2 \end{bmatrix}^{-1} [\xi_i - \xi_j]$$

is the Mahalanobis distance between ξ_i and ξ_j . The characteristic scale of the range representation is set by $\gamma_r^2 = [\gamma_r^2, \dots, \gamma_r^2]$, and similarly for γ_θ^2 and γ_ϕ^2 , where the lengths of the constant vectors correspond to the partitioning of ξ into range and angular components. The overall scale factor γ^2 represents the variance of the estimated function. The kernel parameters can be estimated automatically from the data, using standard fitting routines (Rasmussen & Williams, 2006). It is also possible to constrain the model, e.g. by fixing $\gamma_r = \gamma_\theta = \gamma_\phi$, or $\gamma_\theta = \gamma_\phi$; these simplifications will be evaluated in Section 4.

All applications of kernel (16) to the training data from L training locations can be encoded by an $L \times L$ symmetric matrix \mathbf{K} , with entries

$$K_{ij} = k(\xi_i, \xi_j). \quad (17)$$

The regularized kernel matrix \mathbf{L} is then defined, in the standard way (Rasmussen & Williams, 2006), where σ^2 is the noise variance:

$$\mathbf{L} = \mathbf{K} + \sigma^2 \mathbf{I}. \quad (18)$$

Let Θ and Φ contain the coordinates θ_i and ϕ_i , respectively, of the L training locations, with corresponding RF representations ξ_i . The optimal azimuth and elevation estimates (14) and (15), are obtained from the standard GPR formulas (Rasmussen & Williams, 2006):

$$\theta_*(\xi) = \mathbf{k}(\xi) \mathbf{L}^{-1} \Theta \quad (19)$$

$$\phi_*(\xi) = \mathbf{k}(\xi) \mathbf{L}^{-1} \Phi \quad (20)$$

where ξ contains the newly observed RF data, and the i th entry of the $L \times 1$ vector $\mathbf{k}(\xi)$ is given by $k(\xi_i, \xi)$. It is also possible to compute the conditional probability of each estimate, given the training data (Rasmussen & Williams, 2006). For example,

$$\theta_* | \Theta \sim \mathcal{N}\left(\mathbf{k}(\xi) \mathbf{L}^{-1} \Theta, k(\xi, \xi) - \mathbf{k}(\xi) \mathbf{L}^{-1} \mathbf{k}^T(\xi)\right) \quad (21)$$

where $\mathcal{N}(\mu, \sigma)$ is a 1D Normal distribution, and the analogous formula holds for the elevation estimate, $\phi_* | \Phi$.

3.3. B-spline representation

The angular estimates $[\theta_*, \phi_*]$ could be obtained by applying GP regression directly to the concatenated profiles $[P'(r), P'_A(r_+, \theta), P'_E(r_+, \phi)]$, as described in Section 3.2. In practice, we use lower-dimensional azimuth and elevation representations $[\alpha, \epsilon]$, in Eqs. (14) and (15), with $\xi = [r_+, \alpha, \epsilon]$ being the complete representation, as indicated in Section 3.1. This dimension reduction process improves the performance of the GPR method, as will be shown in Section 4. In addition, there is a computational benefit when constructing the kernel matrix (16), although this is not a major concern in the present context.

We use a B-spline representation, which has the advantages of local support and numerical stability. Let \mathbf{B} be the $M \times N$ matrix of N cubic B-spline basis functions, with optimally located knots (de Boor, 1978). The azimuth and elevation profiles are approximated by $\mathbf{B}\alpha$ and $\mathbf{B}\epsilon$, respectively, where the coefficient column-vectors α and ϵ are estimated by standard least-squares methods. If the rows of \mathbf{B} are denoted by \mathbf{b}_i , then the approximation can be written more explicitly as

$$P'_A(r_+, \theta_i) \approx \mathbf{b}_i \alpha \quad (22)$$

$$P'_E(r_+, \phi_i) \approx \mathbf{b}_i \epsilon \quad (23)$$

where the index $i = 1, \dots, M$ ranges over the angular dimension of the corresponding RF data array. The appropriate dimensionality N of the spline basis \mathbf{B} is an empirical question, which will be addressed in Section 4. Fig. 3 shows an example of dimension reduction for a typical angular profile; in this case the dimensionality is reduced from $M = 63$ to $N = 12$.

4. Localization experiments

This section presents a complete evaluation of the proposed algorithm. Section 4.1 describes the experimental setup, including the relevant radar configuration parameters. Next, Section 4.2 shows how initial target detection can be performed, by background subtraction and direct maximum estimation. Finally, Section 4.3 evaluates several variations of the proposed Gaussian Process approach.

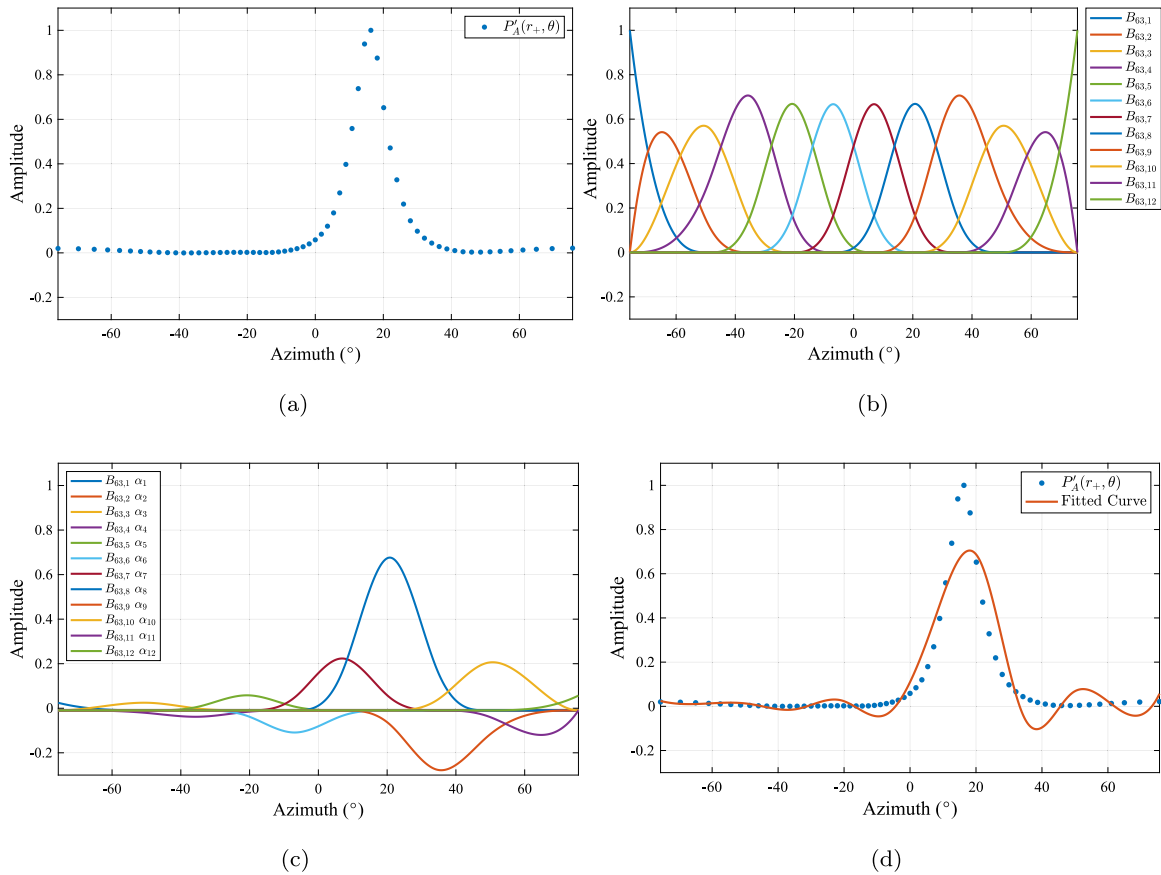


Fig. 3. Representation of an example azimuth profile (a), in a B-spline basis (b). The basis functions (columns of \mathbf{B}) are individually multiplied by the corresponding coefficients α in (c). In this particular example, the profile $P'_A(r_+, \theta)$ goes from 63 samples in (a) to the 12 dimensions of the $\mathbf{B}\alpha$ representation shown in (d).

4.1. Experimental setup

All experiments were performed using a mmWave radar ‘system on a chip’ (IWR6843ISK-ODS ES2.0, Texas-Instruments (2020)), a low power device operating in the 60–64 GHz frequency range, with a $\pm 60^\circ$ field of view, in both azimuth and elevation. The antenna module (Fig. 4) comprises three TXi transmitters, separated by a distance of λ , and four RXi receivers, separated by a distance of $\lambda/2$ (hence a total of 12 virtual antennas). The antenna pattern has relatively low directivity, with 3 dB beamwidth of around 100° in the azimuthal plane and 70° in the elevation plane. Low directivity is beneficial for short-range environments, as this makes a wide field-of-view possible, and high gain is not necessary to overcome signal attenuation or reduced resolution at long distances. The performance of the device has been characterized in detail by the manufacturer, Texas-Instruments (2021). Note that the design allows both azimuth and elevation data to be obtained, given that the signals received at a subset of the antennas (marked with asterisks in Fig. 4) are phase-shifted by 180° . This information is used to form the steering matrix \mathbf{A} , as described in .

The radar device is attached to a movable clamp, together with a digital camera, as shown in Fig. 5. The camera is used to provide estimates of the target ground truth $[\theta, \Phi]$, using the standard Aruco markers (Garrido-Jurado et al., 2016; Romero-Ramírez et al., 2018). Two measures have been taken to improve RF reflection from the MAVs. Firstly, the sensor was mounted above the flight area, pointing down at the body and propellers. A custom aluminium trihedral retro-reflector was also attached to the drone, as can be seen in Fig. 6. The main role of the reflector is to increase the maximum detectable range, in order to enable a more complete evaluation of our proposed algorithms. We used a Parrot Mambo Minidrone, with a wingspan of 13.5 cm, in our experiments, which were conducted in a relatively

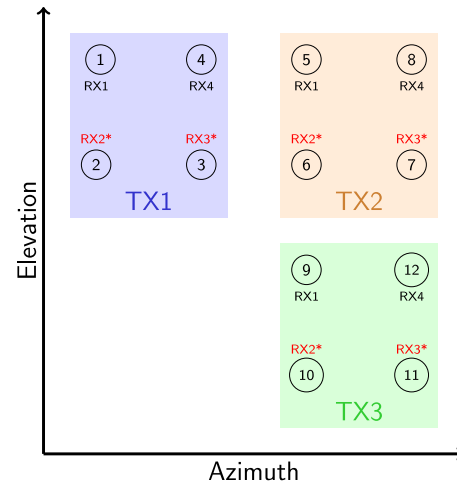


Fig. 4. Antenna layout for the IWR6843ISK-ODS. The steering matrix \mathbf{A} is obtained by considering this reception scheme.

small room (approx. 6m \times 4m), with typical furniture, including shelves, desks, chairs, etc. This environment is relatively challenging, owing to the presence of clutter, and the proximity of reflective walls. The radar was appropriately configured, as summarized in Table 1.

4.2. Preliminary analysis

This section contains the results of our preliminary experiments, which were designed to test the basic localization algorithms developed

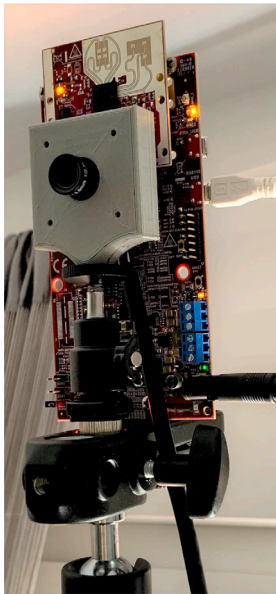


Fig. 5. Experimental device. The antenna module is visible at the top, while the camera is fixed underneath.



Fig. 6. Parrot Mambo Minidrone, with a trihedral retro-reflector for the RCS, for increased RF detectability.

Table 1
Main chirp parameters, adapted to the present experimental setup.

Parameter	Value
Initial frequency	60 GHz
Bandwidth (B)	3.6 GHz
Frequency slope (B/T)	150 MHz/ μ s
ADC resolution	128 bits
ADC sampling frequency	6500 kS/S
Chirps/frame	48

in Section 3.1, using the setup described in Section 4.1. The basic strategy is to estimate the ground truth position of the drone from Aruco markers, which are known to provide reliable estimates (Garrido-Jurado et al., 2016; Romero-Ramírez et al., 2018), assuming that the target is optically visible (which may not be the case, in subsequent

RF applications). We emphasize that the camera system is only used to provide initial reference inputs, for training the Gaussian Process models. The resulting radar system can then operate independently, even in optically opaque environments (or those in which cameras are forbidden, for privacy reasons).

We begin by validating the background subtraction method, described at the beginning of Section 3. We find that this simple method performs well, as shown in Fig. 7. As can be seen, clutter and reflection would mislead the localization process. However, removing static parts of the signal greatly reduces this problem.

As described in Section 3.1, the second way to facilitate target detection is to consider the Doppler range profile, defined in Eq. (8). The corresponding estimate in this equation makes use of the drone bulk velocity, as well as the micro-Doppler effect generated by local motion of the propellers. In particular, the side-lobes also contribute to the main peak in $P'_D(r)$ from Eq. (8), which makes the detection more reliable. Fig. 8 illustrates how this process performs. The peak of the range-azimuth heatmap (Fig. 8a) is around 3m, as shown by the blue profile in Fig. 8c. Meanwhile, the peak of the range-velocity profile, shown in red, is at the correct location of 1m.

We now evaluate the accuracy of the direct maximum approach, described in Section 3.1, using the camera-based ground truth, as described above. A total of $L = 50$ different locations were used, with ranges between 1 and 4 m, azimuth angles between -50° and 50° , and elevation angles between -20° and 20° . These aerial sites cover most of the available flying space. All data were collected in a static environment, in order to make the ground truth acquisition with the camera as reliable as possible. The localization results are shown in Fig. 9, using the spherical $[r, \theta, \phi]$ coordinates that are appropriate for the radar device. It can be seen that the direct range estimates are in accordance with the ground truth, with 5.45 cm root mean square error (RMSE). The angular estimates are less accurate, and a positive bias is apparent in the elevation estimates. This systematic effect can be attributed to miscalibration of the device.

4.3. Regression models

Next we evaluate the accuracy of the GP regression model, developed in Section 3.2. Recall that $(N \times 1)$ B-spline coefficient vectors α and ϵ are computed for the azimuth and elevation profiles at range r_+ . The complete RF representation $\xi = [r_+, \alpha, \epsilon]$ is therefore $(1 + 2N)$ dimensional. We evaluate a range of possible choices, $N = 8, 12, 20$. All of the RF representations ξ_i are randomly divided in two groups: 70% training data and 30% test data. The training group is used to build the GP regression estimator, as described in Section 3.2.

The results are shown in Fig. 10, for comparison with Fig. 9. The red and blue points represent the estimates for training and test data respectively. Firstly, it can be seen that the training data is accurately represented, as would be expected. More importantly, the test estimates are more accurate than those in the direct maximum, as quantified by the RMSE values in Table 2. These values were computed by averaging ten models, created with different random training/test groupings (the figures show example results, from a typical run). In detail, the range, azimuth and elevation errors of the direct maximum method are 5.45 cm, 7.60° and 11.46° , respectively. The proposed GP regression, based on the B-spline representation, achieves substantially lower errors of 3.40 cm, 4.54° and 5.13° . It is clear from Fig. 11 that the GP regression method is able to compensate for the calibration biases that affect the locations of the maxima in Fig. 9. The RMSEs have been highlighted (bold) in Table 2.

We performed additional experiments, to help understand the main results, all of which are shown in Table 2. Apart from the direct maximum strategy already discussed, two kinds of GP regressions are evaluated. Firstly, a **univariate** GPR in which the input for each model is taken separately. These inputs can be the maxima $[r_+]$, $[\theta_+]$, $[\phi_+]$, calculated by the direct strategy. Secondly, a **multivariate** GPR is

Table 2

RMSE for range, azimuth and elevation estimates obtained by applying direct maximum estimation, univariate GPR (only possible for separate inputs) and multivariate GPR for both separate and concatenated inputs, considering the whole profiles or their reduced versions.

	Direct maxima	Univariate GPR		Multivariate GPR							
		$[r_+]$ $[\theta_+]$ $[\phi_+]$	$[P'_R]$ $[P'_A]$ $[P'_E]$	$[\rho]$ $[\alpha]$ $[\epsilon]$	$[P'_R, P'_A, P'_E]$	$[\rho, \alpha, \epsilon]$	$[r_+, \alpha, \epsilon]$				
							$N = 20$	$N = 12$	$N = 8$	$[\gamma_r, \gamma_\theta, \gamma_\phi]$	$\gamma_{r\theta\phi}$
Range (cm)	5.45	3.40	4.61	4.06	4.29	4.06	4.08	4.19	4.20	3.92	4.46
Azimuth (°)	7.60	6.93	5.76	5.78	5.99	5.17	5.20	4.54	5.03	4.79	5.07
Elevation (°)	11.46	5.94	5.73	5.89	6.02	5.71	6.03	5.13	5.42	5.15	6.05

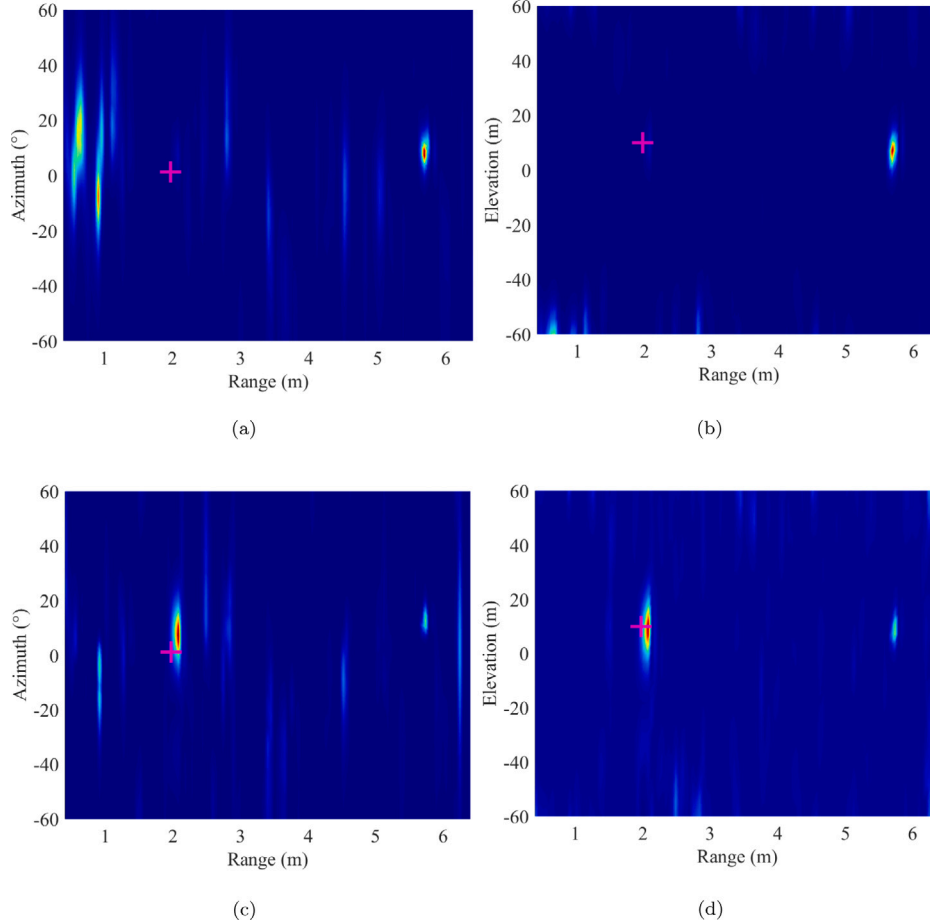


Fig. 7. Background subtraction example for azimuth (a,c), and elevation (b,d) heatmaps. The red cross indicates the true position of the target. Environmental clutter produces false peaks in (a,b), which dominate the target. After background subtraction in (c,d), the dominant peak is close to the true position (estimated by the camera system). The residual biases are modelled by Gaussian Process regression.

applied with different inputs. The first one is carried out with the separate profiles $[P'(r)]$, $[P'_A(r_+, \theta)]$, $[P'_E(r_+, \phi)]$, extracted as proposed in Section 3.1, or their reduced versions $[\rho]$, $[\alpha]$, $[\epsilon]$. A general improvement can be observed for all of them, being noteworthy the one achieved for range. The second one is applied to the concatenated raw profiles, $[P'(r), P'_A(r_+, \theta), P'_E(r_+, \phi)]$, of dimension $[120, 63, 31]$, then the azimuth and elevation RMSEs rise to 5.99° and 6.02° . The method with this input does not provide a clear improvement over the last method. However, if the input consists in a concatenation of the reduced versions $[\rho, \alpha, \epsilon]$ or $[r_+, \alpha, \epsilon]$, then a general RMSE improvement better than for univariate or separate multivariate GPR is found. This suggests that the GP kernel representation works more effectively in the lower-dimensional B-spline space.

Furthermore, we have completed the study by varying the freedom with which the GP routine can optimize the kernel parameters in Eq. (16). The overall variance γ^2 of the estimated function must

be estimated in all cases. The length scales for range, azimuth and elevation can be constrained to a single number $\gamma_{r\theta\phi}$, a pair $[\gamma_r, \gamma_{\theta\phi}]$, or a triple $[\gamma_r, \gamma_\theta, \gamma_\phi]$. The performance of the GP regression was comparable, across these variations, as indicated in Table 2. Overall, the most flexible model performed best, in conjunction with an intermediate spline dimensionality, of $N = 12$.

A summary of the results, plotted in Cartesian coordinates, can be seen in Fig. 12. In most cases, the GPR estimates (blue points) are closer to the ground truth (grey points) than those extracted from the direct maximum method (red points), as expected. The overall RMSEs in space are 22.86 cm for the first method and 31.84 cm for the second.

5. Conclusions

This paper has presented a novel 3D localization system for UAVs, flying in cluttered environments, based on mmWave radar signals.

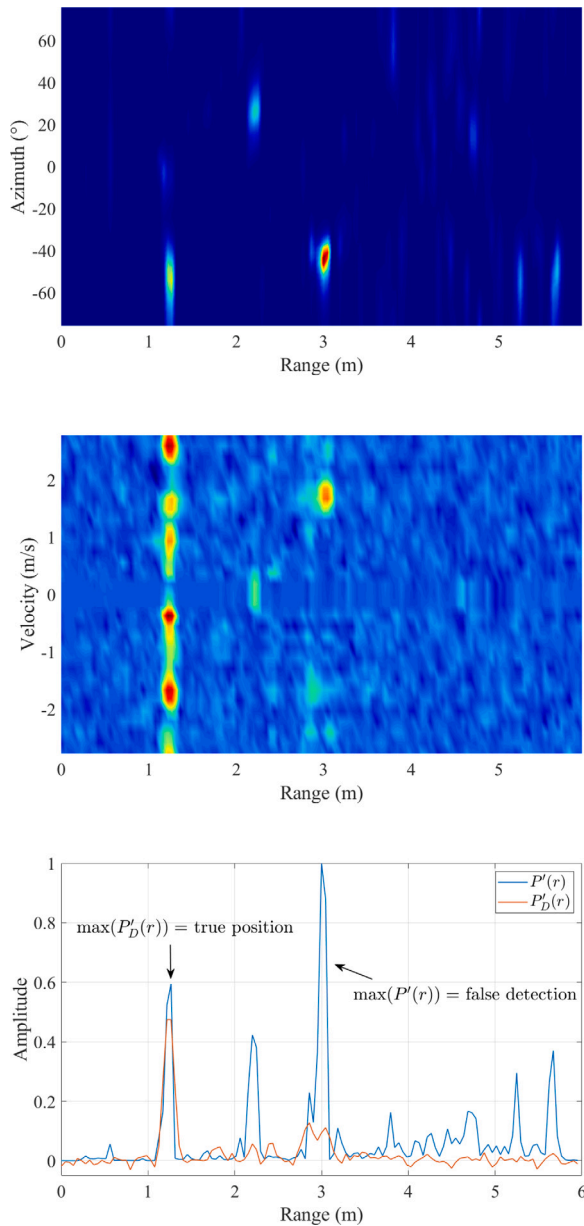


Fig. 8. Example use of micro-Doppler information to assist target detection. The peak of the range profile extracted from the range-azimuth heatmap (top), around 3m, does not correspond to the true target range. However, the peak of the range-velocity data (middle) is located at the true range of the target (bottom).

This is a prototype for a low-cost and low-power system, which could function in poor visibility conditions, such as fog or smoke. The ability to avoid recording conventional images has additional privacy benefits, in urban settings, for example.

We have shown that conventional algorithms, including background subtraction and spectral analysis, can be used to estimate the range of a drone. The additional value of micro-Doppler signals, generated by rotors and propellers, has also been demonstrated. A novel Gaussian Process regression method has been used to refine the range and the angular localization of the target. The importance of performing dimension reduction, before training the regression model, has also been established. The improvement in the RMSE is approximately 3° for azimuth, 6° for elevation, and 2cm in range, with respect to direct maximum methods. Most importantly, the Gaussian Process regression is able to compensate for any mis-calibration of the radar system, which is an important consideration when using commodity devices. Our

experiments also indicate that this approach is robust to environmental clutter. Future work will investigate the performance of these methods in dynamic environments.

CRedit authorship contribution statement

José A. Paredes: Conceptualization, Methodology, Investigation, Writing – original draft. **Fernando J. Álvarez:** Resources, Supervision, Writing – review & editing. **Miles Hansard:** Formal analysis, Supervision, Writing – review & editing. **Khalid Z. Rajab:** Software, Resources, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix. Radar signal processing

The fundamental concept in radar systems is the emission of a signal, and its reception after being reflected from the surrounding objects. Specifically, a frequency modulated continuous wave (FMCW) radar is used in this work, in which a continuous chirp-like signal is employed to synthesize a wide bandwidth, and thus a high range resolution.

The radar working principles are depicted in Fig. A.13. First, a linear chirp-like wave is emitted with a defined period T . Once the echo has been received (delayed version of the emitted wave), the signal passes through an electronic mixer, together with the original version. The output from this mixer is a new signal whose instantaneous frequency comes from the subtraction of the emitted and received instantaneous frequencies, and has a lower frequency f_{IF} component that can be extracted by applying a low pass filtering. This low frequency f_{IF} is directly proportional to the distance r from the radar to the target:

$$r = \frac{cT}{2B} f_{IF} \quad (A.1)$$

where c is the speed of light, and B the bandwidth swept by the emitted chirps. The range resolution is given by:

$$\Delta r = \frac{c}{2B} \quad (A.2)$$

A detailed analysis of an FMCW radar working principle can be read in Richards (2005).

Further target characteristics can be obtained with the emission of two or more time-separated signals. A small change Δd in the distance of an object results in a phase change $\Delta\phi$ in the intermediate signal, which can be expressed as:

$$\Delta\phi = 2\pi f \Delta\tau = 2\pi \frac{c}{\lambda} \frac{2\Delta d}{c} = 4\pi \frac{\Delta d}{\lambda} \quad (A.3)$$

where $\lambda = c/f$ is the wavelength. Thus, velocity v is obtained directly from the phase measurement as:

$$v = \frac{\Delta d}{T_c} = \frac{\lambda \Delta\phi}{4\pi T_c} \quad (A.4)$$

Here T_c is the period between two consecutive chirps. Usually, several chirps are emitted in a certain time interval T_f , known as a frame, to

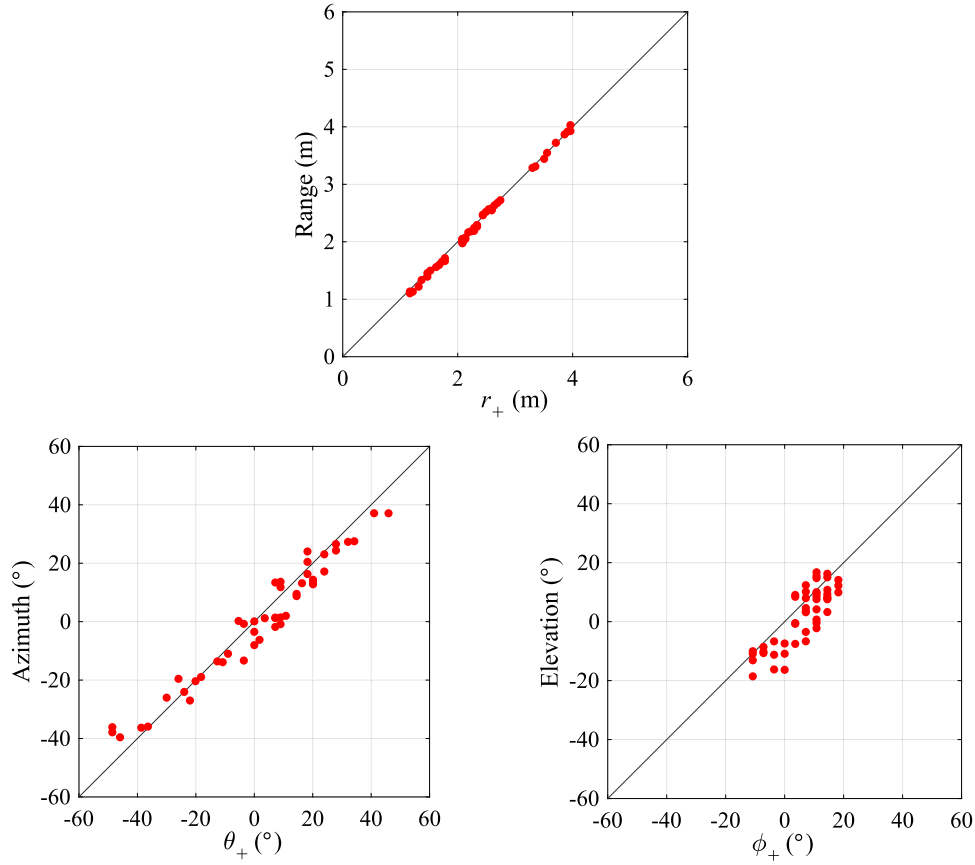


Fig. 9. The horizontal axes represent each coordinate of the estimated position $[r_+, \theta_+, \phi_+]$, obtained from the maxima of the (background subtracted) RF profiles. The vertical axes represent the corresponding coordinates $[R, \theta, \Phi]$, obtained by the camera-based reference method. Errors in the estimates are indicated by vertical deviations from the 45° lines, in each case.

have a more reliable velocity measurement. As will be shown below, this velocity will be used to strengthen the detection of targets in the proposed algorithm.

Finally, extracting azimuth and elevation is necessary in order to obtain the complete set of coordinate information. For this purpose, the device (described in Section 4.1) is based on a multiple-input multiple-output (MIMO) approach (Fishler et al., 2004; Li & Stoica, 2007), which leverages improved processing capabilities to apply digital beamforming for optimizing angular estimation, without the need for expensive phased arrays. Thus, the angle of arrival (AoA) can be computed by making use of multiple receivers:

$$\theta \text{ (or } \phi) = \sin^{-1} \left(\frac{\lambda \Delta \hat{\phi}}{2\pi L} \right) \quad (\text{A.5})$$

where L is the distance between the two receivers, assuming a planar wavefront, and $\Delta \hat{\phi}$ is now the phase difference caused by the delay in the wave arriving at the different receivers. Note that this is a nonlinear dependency, hence the smaller the AoA (closer to 0°), the more accurate the estimates. Moreover, the angular resolution is dependent on the number of antennas K as follows:

$$\Delta \theta \geq \frac{\lambda}{KL \cos \theta} \quad (\text{A.6})$$

In order to extract information about range, velocity, azimuth and elevation, this paper employs spectral analysis techniques, such as fast Fourier transform (FFT) and MUSIC (Schmidt, 1986), through which the response versus range or angle can be estimated, and then the peaks can be found to extract their values. These techniques are reviewed below.

Direct spectral analysis

Firstly, it is necessary to extract f_{IF} from Eq. (A.1), over the total amount of chirps emitted/received during each acquisition period (frame), which provides data about the object range. This operation, which involves an FFT (FFT₁ in Figs. A.14 and A.15), is performed by the device. A second analysis must then be performed, in order to apply Eqs. (A.4) and (A.5) thereby obtaining $\Delta \varphi$ and $\Delta \hat{\phi}$. After the first FFT, all elements of each column in the second matrix, both in Figs. A.14 and A.15, have the same magnitude, while the phases vary linearly. Hence, these phases can be obtained by performing a further spectral analysis, this time over each column. If this is calculated by taking into account the chirps arriving individually at each antenna, then a range-velocity heatmap is obtained (FFT₂ in Fig. A.14). On the other hand, if it is done by arranging the signals according to the pair of antennas that has emitted/received, using a steering vector/matrix (Chen et al., 2010), a range-azimuth or a range-elevation heatmap is produced (FFT₂ or MUSIC in Fig. A.15).

Subspace spectral analysis

A drawback of the FFT approach, when used to extract AoA information, is that the side lobes can mask the main peak. For this reason, a different spectral analysis is applied here: the MUSIC algorithm (Schmidt, 1986), which estimates the frequency content of a signal by using an eigenspace method. The idea is based on decomposing the input signal covariance matrix into signal and noise components:

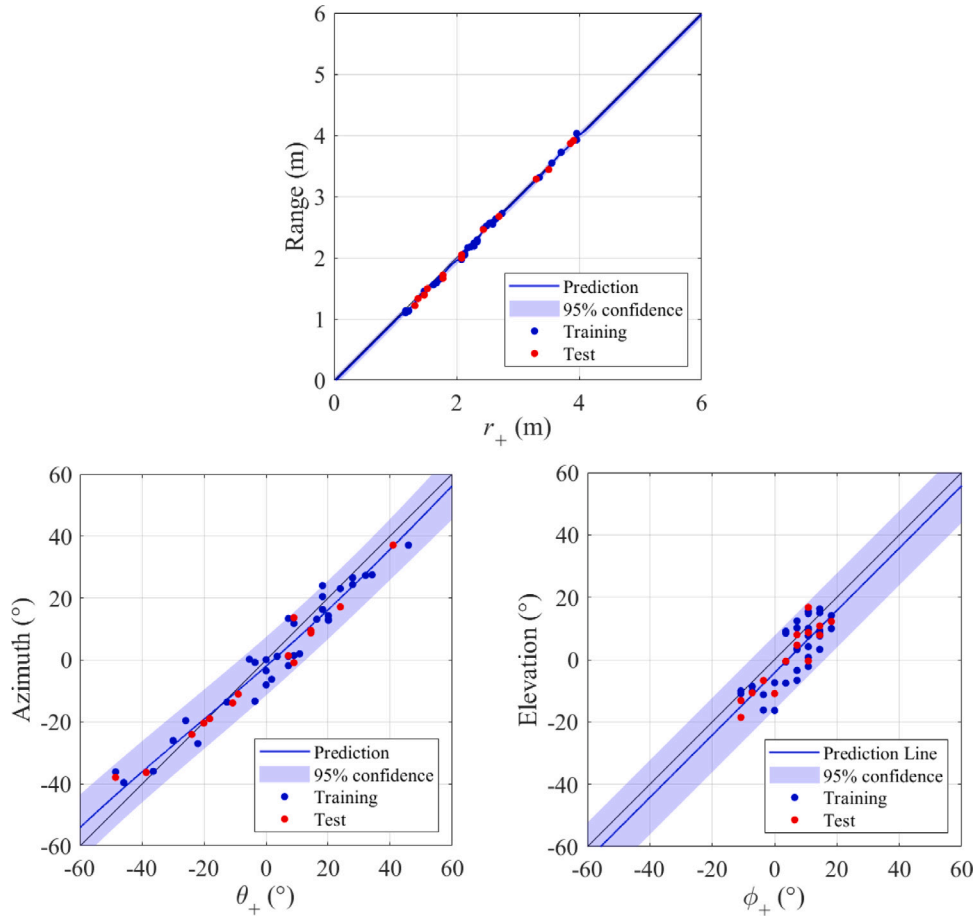


Fig. 10. Gaussian process models of the data in Fig. 9, using independent inputs r_+ , θ_+ , and ϕ_+ , respectively. Errors are represented by vertical deviations of the test points (red) from the estimated function (blue). Note that the azimuth estimate θ_+ (blue curve) is significantly better than the direct maximum estimate θ_+ (45° line). The shaded region is the 95% confidence envelope of the Gaussian Process model.

1. Consider K antennas receiving M -signals \mathbf{x}_i :

$$\mathbf{x}_i(t) = \sum_{m=1}^M s_m(t) e^{j2\pi\tau_m(i-1)} + \boldsymbol{\psi}_i \quad \text{with } 0 < i \leq K \quad (\text{A.7})$$

where s_m is the m th emitted signal out of M , $\tau_m = L \cos(\theta_m)/\lambda$ is the time shift for each antenna, and $\boldsymbol{\psi}_i$ corresponds to noise.

2. The former equation can be expressed in matrix form:

$$\begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_2(t) \\ \vdots \\ \mathbf{x}_K(t) \end{bmatrix} = \begin{bmatrix} \mathbf{a}(\theta_1) & \mathbf{a}(\theta_2) & \dots & \mathbf{a}(\theta_M) \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_M(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{\psi}_1(t) \\ \boldsymbol{\psi}_2(t) \\ \vdots \\ \boldsymbol{\psi}_K(t) \end{bmatrix} \quad (\text{A.8})$$

or, in general:

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \boldsymbol{\Psi} \quad (\text{A.9})$$

3. The $\mathbf{a}(\theta_i)$ are the steering vectors:

$$\mathbf{a}(\theta_i) = [1 \quad e^{j2\pi\tau_1(\theta_i)} \quad e^{j2\pi\tau_2(\theta_i)} \quad \dots \quad e^{j2\pi(K-1)\tau_i(\theta_i)}]^T \quad (\text{A.10})$$

The covariance matrix for \mathbf{X} can be written as:

$$\mathbf{R} = \mathbb{E}[\mathbf{X}\mathbf{X}^H] \quad (\text{A.11})$$

where \mathbb{E} is the operator denoting the expected value (mean) of its argument, and \mathbf{X}^H is the Hermitian transpose of \mathbf{X} . It follows that:

$$\begin{aligned} \mathbf{R} &= \mathbb{E}[(\mathbf{A}\mathbf{S} + \boldsymbol{\Psi})(\mathbf{A}\mathbf{S} + \boldsymbol{\Psi})^H] = \mathbb{S}\mathbb{E}[\mathbf{A}\mathbf{A}^H]\mathbf{S}^H + \mathbb{E}[\boldsymbol{\Psi}\boldsymbol{\Psi}^H] \\ &= \mathbf{S}\mathbf{R}_s\mathbf{S}^H + \sigma^2\mathbf{I} = \mathbf{S}\mathbf{R}_s\mathbf{S}^H + \mathbf{R}_n \end{aligned} \quad (\text{A.12})$$

where \mathbf{R}_n is the noise correlation matrix. The signal correlation matrix \mathbf{R}_s is diagonal and can be expressed as:

$$\mathbf{R}_s = \mathbf{A}\mathbf{A}^H = \begin{bmatrix} \mathbb{E}[|s_1|^2] & 0 & \dots & 0 \\ 0 & \mathbb{E}[|s_2|^2] & \dots & 0 \\ \vdots & \dots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbb{E}[|s_N|^2] \end{bmatrix} \quad (\text{A.13})$$

In practice, \mathbf{R} is unknown, so it must be estimated from the received signal as follows:

1. Estimate the (positive semi-definite) covariance matrix, based on the M received signals:

$$\hat{\mathbf{R}} = \frac{1}{M} \sum_{m=1}^M \mathbf{x}_m \mathbf{x}_m^H \quad (\text{A.14})$$

2. Order the eigenvalues from highest to lowest ($\lambda_1, \dots, \lambda_M, \dots, \lambda_N$), and take the top M (with corresponding eigenvectors) as the signal subspace \mathbf{E}_s . The remaining $N - M$ components constitute the noise subspace \mathbf{E}_n
3. Since the steering vectors corresponding to signal components are orthogonal to the noise subspace eigenvectors, $\mathbf{A}\mathbf{E}_n\mathbf{E}_n^H\mathbf{A} = 0$ for the angles corresponding to AoAs, then the MUSIC spectrum is constructed by taking the following inverse:

$$\mathcal{P}(\theta) = \frac{1}{\mathbf{A}\mathbf{E}_n\mathbf{E}_n^H\mathbf{A}} \quad (\text{A.15})$$

The AoAs of the multiple incident signals can be estimated by locating the peaks in this equation.

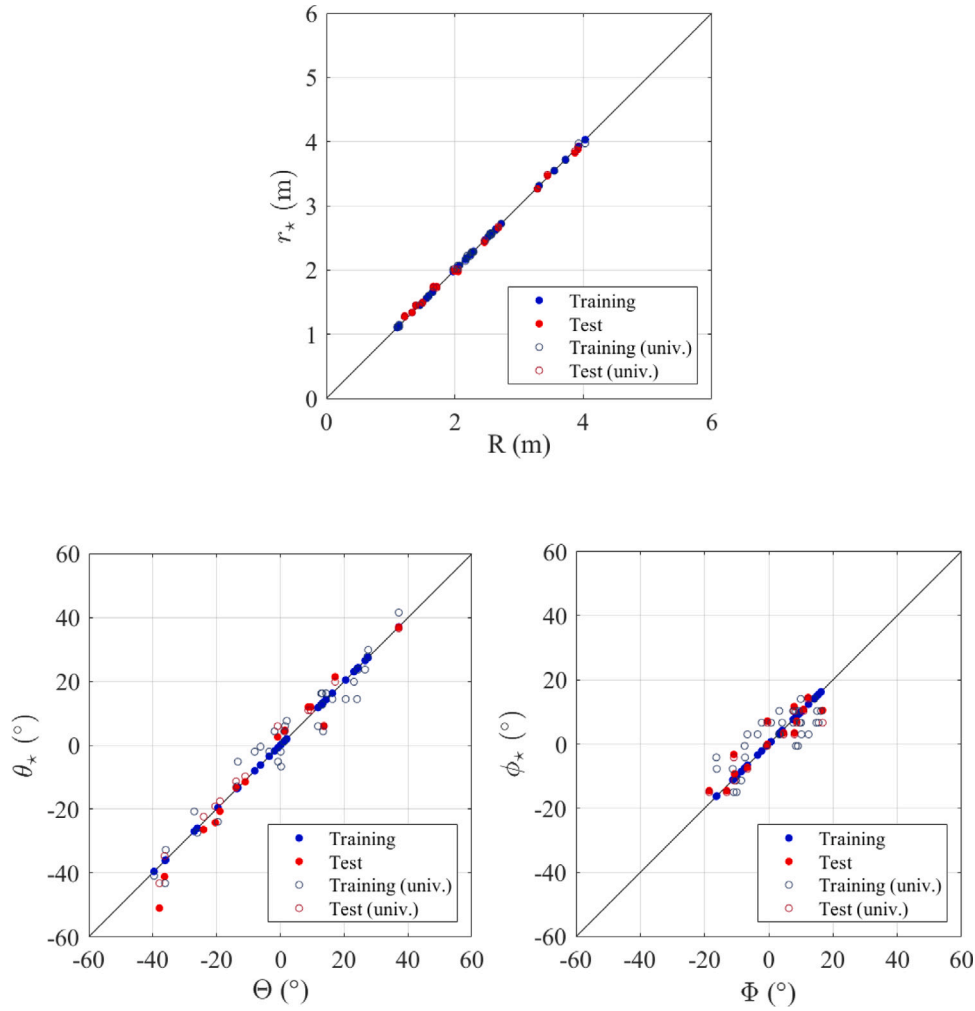


Fig. 11. Results from the complete multivariate model (filled circles). Here the complete multidimensional representations $[r_+, \alpha, \epsilon]$ are used as inputs to each Gaussian Process Regression model. The univariate case results (unfilled circles), where the inputs are taken as $[r_+]$, $[\alpha]$, $[\epsilon]$ individually, are also shown. The horizontal axes show the reference coordinates $[R, \theta, \phi]$, in this figure. The vertical axes show the corresponding estimates, with vertical deviations being prediction errors.

Conceptually, the MUSIC algorithm can achieve high frequency estimation accuracy, but it involves searching for the spectral peak over the full range frequency domain, which is computationally demanding. This fact requires the code to be optimized in terms of execution time.

It is worth noting that we have considered only one moving target, and the presence of a second target complicates the extraction of Doppler characteristics. We note that the beamforming techniques (i.e. MUSIC) presented here can be applied to the MIMO radar outputs, to isolate the Doppler responses of multiple targets. Using Eq. (A.9) a beamformer can be represented as:

$$\mathbf{X}' = \mathbf{W}^H \mathbf{X} = \mathbf{W}^H (\mathbf{A}\mathbf{S} + \mathbf{\Psi}) \tag{A.16}$$

where \mathbf{W} represents the beamforming weight matrix, which is dependent on the AoA and can be calculated using the previously presented techniques as:

$$\mathbf{W} = \mathcal{P}(\theta) \hat{\mathbf{R}}^{-1} \mathbf{A} \tag{A.17}$$

By selecting the beamforming weights to correspond to the AoA of a specific target, and then applying those weights to the Doppler spectrum, it is possible to isolate the Doppler response of a specific target. Furthermore, by properly designing the beamformer weight matrix, the beam can be steered in the desired direction. This is useful because, once the weights have been estimated for the range-angle information, they can be applied to the Doppler response, in order to obtain the

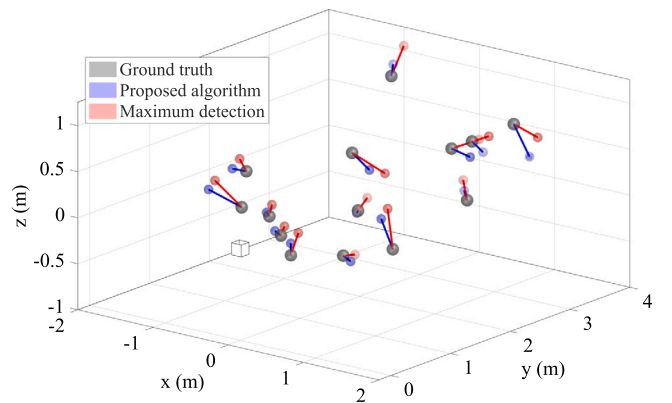


Fig. 12. Summary of results, plotted in physical coordinates. The grey cubes represent the ground truth positions, with the diameter of the spheres representing the drone size. The results from the direct maximum strategy are shown in red, and those from the final proposed GPR method in blue. The line segments show the pattern of residual errors. The small cube indicates the location of the radar device, and the line segments show the pattern of residual errors.

Doppler spectrum for a specific angle. In this paper, only a single target is considered, which simplifies the process.

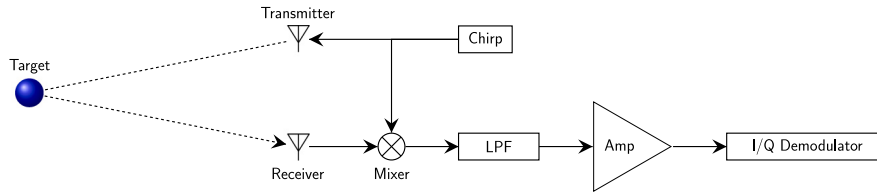


Fig. A.13. Radar operation: First, a wave is transmitted, and once the reflection is received, it is mixed with the generated chirp waveform. The resulting signal is low-pass filtered (LPF), and amplified (Amp). Finally, the I/Q demodulator extracts the real and imaginary components from the IF signal, for input into an ADC.

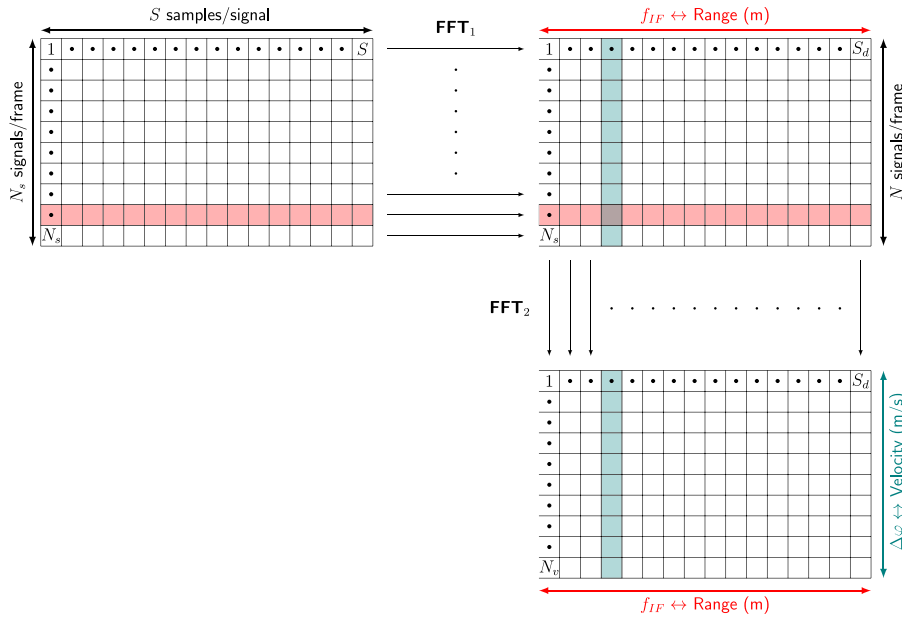


Fig. A.14. Spectral analysis required to get the range-velocity heatmap. At the beginning, every antenna receives N_s signals in a determined period of time (frame). Once all received signals have been organized in an array (top left), an FFT (FFT_1) is performed to obtain f_{IF} , or equally, the range r , according to Eq. (A.1). Finally, the elements of each column for the new array (top right) will have the same magnitude with linearly increasing phase $\Delta\phi$, which can be obtained by applying a new FFT (FFT_2) that will provide velocity information (bottom), as in Eq. (A.4).

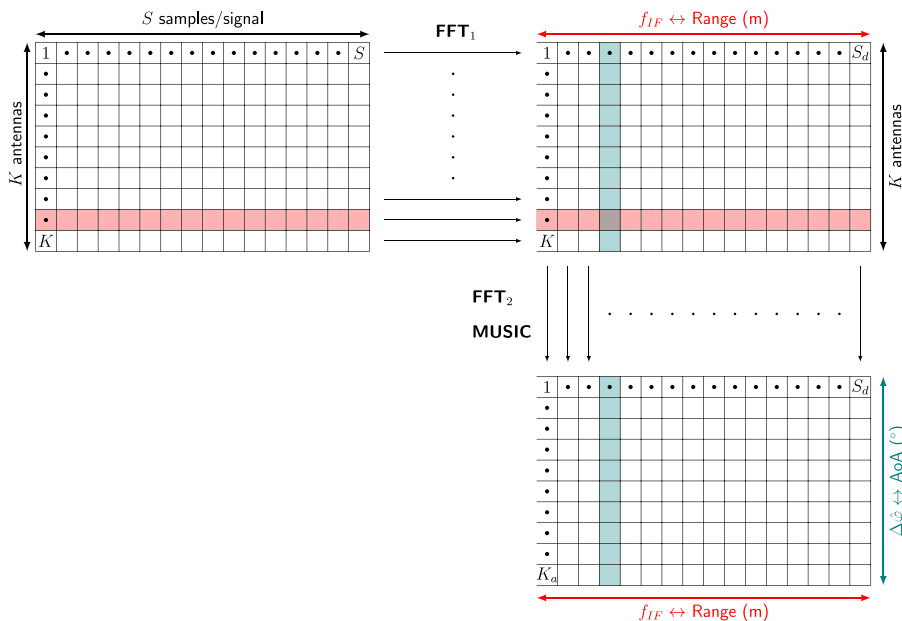


Fig. A.15. Spectral analysis required to get range-AoA (azimuth/elevation) heatmaps. Analogously with the velocity computation, the received signals are arranged in an array, but this time the signals are those received at the same time in different antennas (top left). First, an FFT (FFT_1) can be done to figure out f_{IF} , or equally, the range r -equation (A.1). Finally, the elements of each column for the new array (top right) will have the same magnitude with linearly increasing phase $\Delta\phi$, which can be obtained by applying a new FFT (FFT_2) that will provide angular information (bottom) - equation (A.5).

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